For each question, "E) NOTA" indicates that none of the above answers is correct.

1. Find the volume of Gabriel's horn, which is the solid generated when the region bounded by the function $f(x) = \frac{1}{x}$ and the x-axis for $x \geq 1$ is revolved about the x-axis.

A) π B) $\frac{\pi}{2}$ C) π^2 D) $\frac{\pi^2}{2}$ E) NOTA

2. Evaluate: $\lim_{x \to 0} \left(\frac{sin^{100}x}{x^{99}sin2x}\right)$

A) 0 B) $\frac{1}{2}$ C) 2 D) DNE E) NOTA

3. Suppose that $f(x)$ and $g(x)$ are differentiable functions given $F(x) = \Big(f\big(g(x)\big)\Big)^2$. Given that $g(1) = 2$, $g'(1) = 3$, $f(2) = 4$, and $f'(2) = 5$ find the value of $F'(1)$.

A) 20 B) 40 C) 60 D) 120 E) NOTA

4. Find the volume of the region generated when the area enclosed by $\frac{(x-1)^2}{9} + \frac{(y+2)^2}{4} = 1$ is rotated around the line $x - y - 1 = 0$.

A) $2\sqrt{2}\pi^2$ B) $6\sqrt{2}\pi^2$ C) $6\sqrt{2}\pi$ D) $12\sqrt{2}\pi$ E) NOTA

5. Suppose f is continuous and $x^2 \leq f(x) \leq 6$ for all x in the interval $[-1,2]$. Based directly on this inequality, find values of A and B such that $A \leq \int_{-1}^{2} f(x)dx \leq B$.

A) $A = 3, B = 11$ B) $A = 4, B = 18$ C) $A = 3, B = 18$ D) $A = 4, B = 11$ E) NOTA

6. Among all tangent lines to the graph of $y = \frac{6}{x^2+3}$ at points where $x > 0$, find the tangent line with minimum slope.

A) $y = \frac{3}{4}x + \frac{3}{4}$ B) $y = \frac{3}{4}x + \frac{9}{4}$ C) $y = -\frac{3}{4}x + \frac{3}{4}$ D) $y = -\frac{3}{4}x + \frac{9}{4}$ E) NOTA

Use of the following information for questions 7 and 8.

The concentration $C(t)$ of alcohol in the blood, in %, obeys the decay differential equation $\frac{dC}{dt} = -\frac{1}{k}C$, where $k = 2.5$ hours and is called the elimination time. It is estimated that a male weighing 70 kilograms who drank 3 pints of beer over a period of one hour has a concentration of 1% alcohol in his blood. The allowed legal concentration for driving is a maximum of 0.5%.

7. If a 70-kilogram male has a blood alcohol concentration of 1%, how many hours should he wait before driving in order not to disobey the law?

A) 2.5 B) $2.5 \ln 2$ C) $\ln 2$ D) $1/2$ E) NOTA

8. What is the initial (t=0) rate of change of the blood alcohol concentration with respect to time for this male?

A) $-1/250$ B) $-2/125$ C)$-5/2$ D) $-1/125$ E) NOTA

9. Given that $\sin^0 x = 1$, evaluate the following expression:

$$\sum_{k=0}^{\infty} \int_0^{\frac{\pi}{3}} \sin^{2k} x \, dx$$

A) $\frac{\sqrt{3}}{3}$ B) $\sqrt{3} - 1$ C) $\sqrt{3}$ D) ∞ E) NOTA

10. Consider the tetrahedron ABCD with the vertices $A = (a, 0,0), B = (0, b, 0), C = (0,0, c)$, and $D = (0,0,0)$. Additionally the areas of DAB, DBC, and DCA are 3, 14, and 18 respectively. What is the volume of ABCD?

A) $\sqrt{21}$ B) $6\sqrt{21}$ C) $2\sqrt{42}$ D) $12\sqrt{42}$ E) NOTA

11. Given that $y = \frac{\sqrt{1+2x}\sqrt[4]{1+4x}\sqrt[6]{1+6x}\ldots\sqrt[2n]{1+2nx}\ldots\sqrt[100]{1+100x}}{\sqrt[3]{1+3x}\sqrt[5]{1+5x}\sqrt[7]{1+7x}\ldots\sqrt[2n+1]{1+(2n+1)x}\ldots\sqrt[101]{1+101x}}$ find $\frac{dy}{dx}$ at $x = 0$.

A) 1 B) $\frac{100!}{101!-1}$ C) $\frac{100^{100}}{101!}$ D) $\frac{3*5*7*\ldots*101}{2*4*6*\ldots*100}$ E) NOTA

12. Find the number of integral solutions to the inequality $f'(x) < f(x)$ where $f(x) = e^{x^3-5x^2-7x+9}$.

A) 3 B) 4 C) 5 D) 6 E) NOTA

13. Suppose that we do not have a formula for $g(x)$ but we know that $g(2) = -4$ and $g'(x) = \sqrt{x^2+5}$ for all x. Use a linear approximation to estimate $g(2.05)$.

A) -2.5 B) -5.5 C) -3.85 D) -4.15 E) NOTA

14. Evaluate the following integral:

$$\int_0^{\infty} \frac{d}{dx}[e^{1+x-x^2}]\,dx$$

A) $-e$ B) e C) 1 D) Divergent E) NOTA

15. Suppose that a and b are real numbers such that $\lim_{x\to 0} \frac{\sin^2 x}{e^{ax}-bx-1} = \frac{1}{2}$. There are two ordered pairs (w, x), (y, z) such that the limit exists. Find $w + x + y + z$.

A) 0 B) 2 C) 4 D) 6 E) NOTA

16. A movie screen on a wall is 20 feet high from bottom of screen to top of screen, and the bottom of the screen is 10 feet above the floor. At what distance x, in feet, from the front of the room should you position yourself so that the viewing angle θ (this corresponds to the angle between the line of sight to the bottom of the screen and the top of the screen) of the movie screen is as large as possible?

A) $5\sqrt{2}$ B) $10\sqrt{2}$ C) $5\sqrt{3}$ D) $10\sqrt{3}$ E) NOTA

17. Evaluate the following sum:

$$\sum_{n=0}^{\infty} \frac{(-1)^n(\log_{2016} 2)^n}{n!\,(\log_{2016} e)^n}$$

A) 1/2 B) 2 C) 2016 D) e E) NOTA

18. Find the volume of the solid whose base is bounded by the circle $x^2 + y^2 = 1$ and whose cross-sections perpendicular to the base and perpendicular to the x-axis are squares.

A) $\frac{256}{3}$ B) $\frac{144}{3}$ C) $\frac{128}{3}$ D) $\frac{192}{3}$ E) NOTA

19. The solution to the following definite integral

$$\int_{\sin 1}^{\csc 1} \frac{\sin^3(\log_3 x)}{x}\,dx$$

can take the form: $A\cos^3(\log_3 x) + B$, for real numbers A and B. What is $A + B$?

A) 0 B) 5 C) 7 D) 10 E) NOTA

20. $y = \dfrac{9}{x+\sqrt{x+\dfrac{9}{x+\sqrt{x+\frac{9}{x+\ldots}}}}}$. At $x = 1$ what is $\frac{dy}{dx}$?

A) -1 B) 0 C) 1 D) 2 E) NOTA

21. Sand is pouring out of a tube at $1\ m^3/s$. It forms a pile which always maintains the shape of a right circular cone. The height of the cone is equal to the radius of the circle at its base at any time. How fast is the sand pile rising when it is $2\ m$ high?

A) $\frac{1}{4\pi}$ B) $\frac{1}{\pi}$ C) $\frac{1}{3\pi}$ D) $\frac{1}{2\pi}$ E) NOTA

22. What is the area swept out by the spiral $r = e^{\theta}$ from $\theta = 0$ to $\theta = 2\pi$?

A) $\frac{1}{4}e^{4\pi}$ B) $\frac{1}{2}e^{4\pi} - 1$ C) $\frac{1}{4}e^{4\pi} - \frac{1}{4}$ D) $\frac{1}{2}e^{4\pi} - \frac{1}{2}$ E) NOTA

23. Let $f(x) = x^4 + 4x^3 + 8x^2 + 9x + 10$ have roots a, b, c, and d. Find

$$\frac{a^2}{b^2+c^2+d^2} + \frac{b^2}{a^2+c^2+d^2} + \frac{c^2}{a^2+b^2+d^2} + \frac{d^2}{a^2+b^2+c^2}.$$

A) -9 B) -8 C) -4 D) 0 E) NOTA

24. $f(x) = \begin{cases} x^2 - ax + b: & x \le a \\ x^2 - bx + a: & x > a \end{cases}$ is continuous, where a and b are distinct real numbers, find the value of a.

A) -2 B) -1 C) 0 D) 2 E) NOTA

25. Evaluate the following limit: $\lim_{n\to\infty} \ln\left(\frac{n!}{n^n}\right)^{1/n}$

A) 0 B) -1 C) 1 D) does not exist E) NOTA

26. With a straight piece of wire 4 meters long, you are to create either (a) an equilateral triangle and a square by cutting the wire into two pieces, or (b) either one only without cutting the wire. Suppose a piece of length x meters, where $x < 4$, is bent into a triangle and the remainder, if any, is bent into a square. Find the value of x which maximizes the total area of both the triangle and the square.

A) 0 B) 2 C) 4 D) $2\sqrt{2}$ E) NOTA

27. The domain of $y = f(x)$ is [0,6]. Find the domain of $g(x) = (2x - 1)f(3x - 6)$.

A) $[0,2]$ B) $[0,4]$ C) $[0,12]$ D) $[2,4]$ E) NOTA

28. Let $f(x) = x^{2008} + 3x^2 + 5x + 2$ and $F(x) = f(x) + f'(x) + f''(x) + \cdots + f^{(2008)}(x)$. Find the derivative of $g(x) = e^{-x}F(x)$ at $x = 1$.

A) $\frac{11}{e}$ B) $-11e$ C) 0 D) $-\frac{11}{e}$ E) NOTA

29. Approximate the value of $\int_0^1 \frac{\sin x}{x} dx$ by using a Maclaurin polynomial of degree three for the numerator of the integrand.

A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) $\frac{5}{6}$ D) $\frac{17}{18}$ E) NOTA

30. What is the area enclosed by the graph $|x| + |y| = 4$?

A) 16 B) $16\sqrt{2}$ C) 32 D) $32\sqrt{2}$ E) NOTA