Remember—NOTA means "None of the Above"

1. Zach has an appointment today at 11 AM. He normally drives at 45 mph and the drive from his house to the appointment takes 40 minutes. If he is running late, and leaves his house at 10:30 (instead of 10:20), how fast should does he need to drive to get to the appointment on time?

   a) 50 mph   b) 58 mph   c) 60 mph   d) 62 mph   e) NOTA

2. Two buses start 60 miles apart and they move directly towards each other at 20 mph (and continue this speed throughout). A fly starts exactly in front of one bus and flies in a straight line to the other bus, then back to the first bus (again in a straight line), and continues this way until the busses collide and the fly is smashed. If the fly flies at 30 mph, how far does it fly (in miles) before the busses collide?

   a) 30   b) 35   c) 40   d) 45   e) NOTA

3. A train moves past a telephone pole (with negligible width) and a bridge 264 m long in 8 seconds and 20 seconds respectively. What is the speed of the train (in km/hr)?

   a) 69.5   b) 70   c) 79   d) 79.2   e) NOTA

4. The velocity function of a moving particle on a coordinate line is $v(t) = t^3 - 6t^2 + 11t - 6$. Find the total distance traveled by the particle during $1 \le t \le 4$.

   a) 7/4   b) 9/4   c) 11/4   d) 13/4   e) NOTA

5. Three towns A, B and C, form an equilateral triangle with sides of length 300 miles. Plane 1 flies back and forth from Town A to Town B beginning at Town A at 12 noon and continuing until 4 PM. This plane flies at 400 mph and stays on the ground for 15 minutes for refueling at each town. Plane 2 flies in alphabetical order, cycling back to town A from town C and repeating the order, beginning at Town A at 12 noon and continuing until 3:45 PM. This plane flies at 600 mph and stays on the ground for 15 minutes for refueling at each town. Every day, when the times match, the pilots of the 2 planes meet for a cup of coffee while their planes are refueling. What is the only time in the afternoon that this can happen (the beginning time of the refueling)?

   a) 1:30 PM   b) 1:45 PM   c) 2:00 PM   d) 2:30 PM   e) NOTA

6. When not on a moving sidewalk, Catherine can walk the length of the sidewalk in 3 minutes. If she stands on the sidewalk as it moves, she can travel the length in 2 minutes. If Catherine walks on the sidewalk as it moves in the direction it moves, how many minutes will it take her to travel the same distance? Assume she always walks at the same speed.

   a) 1   b) 1.2   c) 1.4   d) 1.6   e) NOTA

7. Two people stand back to back next to the rails in a small railway station. As the head of the express train that passes the station reaches them, they start to walk parallel to the rails (in opposite directions). As the tail of the train reaches each of them, they stop, having walked 30 m and 40 m, respectively. If they both walked with identical, constant speed, and if the train maintained its speed as well, how long is the train?

a) 200 m   b) 240 m   c) 280 m   d) 300 m   e) NOTA

8. An athlete covers three consecutive miles by swimming the first, running the second and cycling the third. He runs twice as fast as he swims and cycles one and a half times as fast as he runs. He takes ten minutes longer than he would if he cycled the whole three miles. How many minutes does he take?

a) 16   b) 22   c) 30   d) 46   e) NOTA

9. An army is moving in a convoy that stretches for 3 miles. Observing radio silence, the general at the very end of the convoy sends a message to the very front via a special runner. After delivering the message, the runner returns to the general. Both the convoy and the runner travel at different, but constant rates. If the entire convoy travels 6 miles in the time it takes for the runner to travel to the very front and return to the very end, what is the total distance in miles that the runner travels?

a) $5\sqrt{3}$ b) $12 - 3\sqrt{3}$ c) $3 + 3\sqrt{5}$ d) $6\sqrt{3}$ e) NOTA

10. On a trip from A to Z, Sonny had to make a detour to pass thru M (without stopping). Forty minutes after he left A, he noted that the remaining distance to M was twice as much as he had traveled so far. After traveling another 21 miles, he calculated that the remaining distance to Z was twice as much as what he had left to get to M. He arrived in Z an hour and a half later. If he traveled at a constant speed, how far was this trip from A to Z (in miles)?

a) 99   b) 108   c) 112   d) 127   e) NOTA

11. A straight pine tree is 14 feet high and a bird is sitting at the very top of it. The wind blows away a feather of the bird. The feather moves uniformly along a straight line at the speed of 4 feet per second. It falls on the ground 4.5 seconds later at a distance D from the base of the pine tree. Which of the following intervals contains the number D?

a) (0, 10]   b) (10, 11]   c) (11, 12]   d) (12, 13]   e) NOTA

12. Three ants are sitting at the three corners of an equilateral triangle. Each ant starts randomly, picks a direction and starts to move along the edge of the triangle. In the worst case scenario, no collisions, the ants stop at the first vertex they reach. What is the probability that none of the ants collide?

a) 1/32   b) 1/16   c) 1/8   d) 1/4   e) NOTA

13. Let A be the point (7, 4) and let D be the point (5, 3). What is the length of the shortest path ABCD, where B is a point $(x, 2)$ and C is a point $(x, 0)$? This path consists of 3 connected segments, with the middle one vertical.

a) $2 + \sqrt{29}$ b) $\sqrt{31}$ c) $2 + \sqrt{31}$ d) $2 + \sqrt{33}$ e) NOTA

14. Donna and her dog walk home from a park by the lake. It takes Donna 36 minutes to walk home and her dog walks twice as fast. They start together, but the dog reaches home before Donna and returns to go back and meet Donna. After greeting Donna, the dog walks home again at double Donna's speed, and then turns back to greet Donna again (and does so). Donna starts at noon to walk home. If Donna and her dog maintain the same speed throughout, how many minutes later does she meet the dog for the 2nd time?

a) 24   b) 27   c) 30   d) 32   e) NOTA

15. A cyclist rides his bike over a route which is 1/3 uphill, 1/3 on level ground and 1/3 downhill. If he covers the uphill part at the rate of 16 miles per hour (mph) and the level part at 24 mph, at what rate in miles per hour would he have to travel the downhill part of the route in order to average 24 mph for the entire route?

a) 32   b) 36   c) 40   d) 44   e) NOTA

16. An insect lives on the surface of a regular tetrahedron with edges of length 1. It wishes to travel on the surface of the tetrahedron from the midpoint of one edge to the midpoint of the opposite edge. What is the length of the shortest such trip? (Note: Two edges of a tetrahedron are opposite if they have no common endpoint.)

a) $\frac{1}{2}\sqrt{3}$ b) 1   c) $\sqrt{2}$ d) $\frac{3}{2}$ e) NOTA

17. Three points are chosen randomly and independently on a circle. What is the probability that all 3 pairwise distances between the points are less than the radius of the circle?

a) 1/36   b) 1/24   c) 1/18   d) 1/12   e) NOTA

18. A particle moves along a curve so that its position at time t is given by the position vector $\left\langle 4e^{3(t-1)}, \cos(t-1) \right\rangle$. What is the speed of the particle when $t = 1$?

a) 7   b) $e^2 + 1$ c) 12   d) $e^3 - 1$ e) NOTA

19. A particle moves along the line $y = 2x + 7$. What is its minimum distance from the origin?

a) $\frac{7}{5}$ b) $\frac{7\sqrt{5}}{5}$ c) $\frac{14}{5}$ d) $\frac{14\sqrt{5}}{5}$ e) NOTA

20. Moving only on the edges of an icosahedron, determine the number of shortest paths from a given vertex A to the vertex opposite A.

    a) 10   b) 15   c) 20   d) 25   e) NOTA

21. Let P equal the parabola with vertex at the origin and directrix y = -1. Determine the number of lattice points on P whose distance from (0, 1) is less than or equal to 197.

    a) 14   b) 21   c) 28   d) 35   e) NOTA

22. The vertices of a regular dodecagon are given by $\left(x_i, y_i \right)$ for $i = 1, 2, ..., 12$ in clockwise order. If

    $\left(x_1, y_1 \right) = (15,9)$ and $\left(x_7, y_7 \right) = (15,5)$, compute $\sum_{i=1}^{12}(x_i - y_i)$.

    a) 56   b) 72   c) 67   d) 96   e) NOTA

23. Two-thousand overlapping square playing cards of width 2" are placed in a row such that one pair of opposite edges of the playing cards are always collinear, overlapping vertically. The 2nd card overlaps the 1st card by 1/1000 of an inch and each successive card overlaps the previous card by one-thousandth of an inch **more**. Determine the longer length of the pile of cards in inches.

    a) 1999   b) 2000   c) 2001   d) 2002   e) NOTA

24. Starting at 11:44 AM, Sebby walked for 5 miles. He noticed that his average reciprocal speed (in minutes and seconds per mile) was numerically equal to the time at which he stopped if he thought of the hour as minutes and the minutes as seconds. Determine the time at which he stopped (in the same time zone).

    a) 12: 38 PM   b) 12:48 PM   c) 12:58 PM   d) 1:08 PM   e) NOTA

25. The graph of the function $f(x) = Sin^{-1}(2\sin x)$ consists of the union of disjoint pieces. Find the distance between the endpoints of any one piece.

    a) $\dfrac{\pi}{3}$ b) $\dfrac{\pi\sqrt{2}}{3}$ c) $\dfrac{\pi\sqrt{5}}{3}$ d) $\dfrac{\pi\sqrt{10}}{3}$ e) NOTA

26. A spike is hammered into a train rail. You are standing at the other end of the rail. You hear the sound of the hammer strike both through the air and through the rail itself. These sounds arrive at your point six seconds apart. You know that sound travels through air at 1100 feet per second and through steel at 16,500 feet per second. How far away is that spike in feet?

    a) $\dfrac{2750}{7}$ b) $\dfrac{23750}{7}$ c) $\dfrac{49500}{7}$ d) $\dfrac{123750}{7}$ e) NOTA

27. If you have gone 20 feet/second for 4 minutes, how far have you gone in inches?

   a) 2800   b) 14400   c) 28800   d) 57600   e) NOTA

28. If you have gone 1200 inches at 20 feet/hour, how much time has elapsed in hours?

   a) 2   b) 2.5   c) 4   d) 5   e) NOTA

29. It takes 10 minutes for a canoeist to paddle perpendicular to the current across a river of uniform width. (Of course, while crossing the river the canoeist drifts downstream.) The canoeist then paddles upstream for 50 minutes, re-crosses the river and paddles downstream for 20 minutes (this time is for the downstream paddling only), returning to the starting point. What is the ratio of the canoeist's speed in still water to the speed of the river current?

   a) 2   b) 2.5   c) 3   d) 3.5   e) NOTA

30. To the nearest thousand feet, what is the distance above sea level (elevation) of the base of the Park City Mountain Resort?

   a) 5000   b) 6000   c) 7000   d) 8000   e) NOTA