

For all questions, answer choice “E) NOTA” means that none of the above answers is correct.

1. A particle moves along a line such that its velocity is given by  $v(t) = 9t^2 - 8t + 5$ . Find the particle's displacement between times  $t = 1$  and  $t = 2$ .

- A) 14                      B) 16                      C) 17                      D) 19                      E) NOTA

2. Let  $k$  be a real number. If  $\int_{-1}^k (3x^2 - 2x) dx = 6$ , then  $k =$

- A) -1                      B) 0                      C) 1                      D) 2                      E) NOTA

3. If  $\int_1^3 f(x) dx = 5$  and  $\int_1^7 f(x) dx = -4$ , then compute  $\int_7^3 (x + f(x)) dx$ .

- A) -21                      B) -11                      C) -1                      D) 9                      E) NOTA

4.  $\int_0^4 (|x - 2| + 3) dx =$

- A) 7                      B) 12                      C) 14                      D) 16                      E) NOTA

5. Which one of the following is an antiderivative of  $\frac{6x^4 + 3x^2 + 1}{x^2}$ ?

A)  $\frac{18x^4 + 15x^2 + 15}{5x^2}$

B)  $\frac{2x^4 + 3x^2 - 1}{x}$

C)  $\frac{12x^4 - 2}{x^3}$

D)  $\frac{18x^4 + 5x^2 + 5}{x}$

E) NOTA

6.  $\int xe^{-x} dx =$

- A)  $e^{-x}(1 - x) + C$     B)  $\frac{e^{1-x}}{1-x} + C$     C)  $-e^{-x}(x + 1) + C$     D)  $-\frac{1}{2}x^2e^{-x} + C$     E) NOTA

7.  $\int x \sinh(x^2) dx =$

A)  $-\frac{1}{2} \cosh(x^2) + C$

B)  $\frac{1}{2} \cosh(x^2) + C$

C)  $x \cosh(x^2) - \sinh(x^2) + C$

D)  $x \cosh(x^2) + \sinh(x^2) + C$

E) NOTA

8.  $\int_0^1 \int_1^{e^x} (x + y) dy dx =$

A)  $\frac{e^2 - 1}{4}$

B)  $\frac{e^2 - 1}{2}$

C)  $e^2 - 1$

D)  $\frac{e^2 + 1}{2}$

E) NOTA

9. The length of the curve  $C$  from  $x = 2$  to  $x = 3$  is given by

$$\int_2^3 \sqrt{1 + (12x^2 - 4)^2} dx.$$

Which of the following is the equation of  $C$ , given that  $C$  goes through the point  $(2, 22)$ ?

A)  $y = 4x^3 - 4x + 2$

B)  $y = 4x^3 - 4x - 2$

C)  $y = 12x^2 - 4$

D)  $y = 12x^2 - 94$

E) NOTA

10. Let  $\lfloor x \rfloor$  be the greatest integer less than or equal to  $x$  and let  $\lceil x \rceil$  be the least integer greater than or equal to  $x$ . Compute  $\int_0^{10} (\lceil x \rceil - \lfloor x \rfloor) dx$ .

A) 0

B) 9

C) 10

D) 11

E) NOTA

11. Let  $a$  and  $b$  be integers. If  $\int_0^6 (ax + b) dx = 8$ , then

- A)  $a$  must be even.
- B)  $a$  must be odd.
- C)  $b$  must be even.
- D)  $b$  must be odd.
- E) NOTA

12. A continuous function  $f$  has values shown in the table below, where  $a$ ,  $b$ , and  $c$  are real numbers.

$x$	0	1	2	3	4
$f(x)$	$a$	$b$	$c$	$c$	$a$

Let  $T_2$  be the trapezoidal sum approximation of  $\int_0^4 f(x) dx$  with 2 equal subintervals. Let  $T_4$  be the trapezoidal sum approximation of  $\int_0^4 f(x) dx$  with 4 equal subintervals. Compute  $T_2 - T_4$ .

- A)  $a - b$
- B)  $a + b$
- C)  $a + b + c$
- D)  $b + 2c$
- E) NOTA

13. Let  $F(x) = \int_1^x \frac{1}{t} dt$ . Find the average value of  $F$  over the interval  $\left[\frac{1}{3}, 3\right]$ .

- A)  $-1 + \ln 3$
- B)  $\frac{3}{4} \ln 3$
- C)  $\ln 3$
- D)  $\frac{8}{3} \ln 3$
- E) NOTA

14.  $\int_1^9 \frac{1}{4 + 5\sqrt{x}} dx =$

- A)  $\frac{16}{25}(1 - \ln 3)$
- B)  $\frac{4}{25} \left( 5 + 2 \ln \frac{9}{19} \right)$
- C)  $\ln \frac{19}{9}$
- D)  $2 + \frac{4}{5} \ln \frac{9}{19}$
- E) NOTA

15. Find the area of the region bounded by the curve  $y = \frac{8}{x}$  and the line  $y = 16 - 2x$  for  $e \leq x \leq e^2$ .

- A)  $17e^2 - e^4 - 16e - 8$   
 B)  $16e^2 - 16e - 8$   
 C)  $16e^2 - e^4 - 16e - 8$   
 D)  $16e^2 - e^4 - 16e + 8$   
 E) NOTA

16. Find the average value of  $g(x) = 4^{\sinh x} \cosh x$  over the interval  $[0, 3]$ .

- A)  $\frac{4^{\sinh 3}}{3 \ln 4}$       B)  $\frac{4^{\sinh 3} - 1}{3 \ln 4}$       C)  $\frac{4^{\sinh 3}}{\ln 4}$       D)  $\frac{1}{3 \ln 4}$       E) NOTA

17.  $\int_2^3 \frac{1}{x^2 - 4x + 5} dx =$

- A)  $\frac{1}{2} \ln \frac{2}{5}$       B)  $\frac{1}{6} \ln \frac{1}{2}$       C)  $\frac{1}{6} \ln 2$       D)  $\frac{\rho}{4}$       E) NOTA

18.  $\int_0^1 e^{e^x+x} dx =$

- A)  $e^{-1}$       B)  $e^e - e$       C)  $e^e + 1$       D)  $e^{e+1}$       E) NOTA

19. What are all values of  $p$  for which  $\int_1^{\infty} \frac{1}{x^{2p-3}} dx$  diverges?

- A)  $p \notin 0$       B)  $p \notin 1$       C)  $p \notin \frac{3}{2}$       D)  $p \notin 2$       E) NOTA

20.  $\int_0^1 \frac{x^5 + 5x^4 + 10x^3 + 10x^2 + 4x}{(x+1)^5} dx =$

- A)  $\frac{1}{8}$       B)  $\frac{17}{24}$       C)  $\frac{49}{64}$       D)  $1 + 4 \ln 2$       E) NOTA

21. Let  $R$  be the region bounded by  $y = e^{-x^2}$  and the  $x$ -axis over  $[0, \sqrt{\ln 10}]$ . Find the volume of the solid generated when  $R$  is revolved about the  $y$ -axis.

- A)  $\frac{9\rho}{20}$       B)  $\frac{11\rho}{20}$       C)  $\frac{9\rho}{10}$       D)  $\frac{11\rho}{10}$       E) NOTA

22. The integral  $\int_{2/\sqrt{3}}^2 \sec^{-1} x \, dx$  is

- A) equal to  $\frac{2\pi}{3} - \frac{\pi}{3\sqrt{3}} + \ln|2\sqrt{3} - 3|$ .  
 B) equal to  $\frac{\pi}{3} - \frac{2\pi}{3\sqrt{3}} + \ln|2\sqrt{3} - 3|$ .  
 C) equal to  $\frac{\pi}{3} - \frac{2\pi}{3\sqrt{3}} + \ln|2\sqrt{3} + 3|$ .  
 D) undefined.  
 E) NOTA

23.  $\int_{\sqrt{\rho}}^{\sqrt{2\rho}} x^3 \cos(x^2) \, dx =$

- A) 0      B) 1      C)  $\frac{\rho}{2}$       D) 2      E) NOTA

24. Find the area inside the loop of the polar curve defined by

$$r = \begin{cases} 5(\sec \theta + \tan \theta) & \frac{\pi}{2} < \theta < \frac{3\pi}{2}, \quad \frac{3\pi}{2} < \theta < \frac{5\pi}{2} \\ 0 & \theta = \frac{3\pi}{2} \end{cases}$$

- A)  $\frac{5}{2}(4 - \rho)$       B)  $\frac{5}{2}(\rho - 2)$       C)  $\frac{25}{2}(4 - \rho)$       D)  $\frac{25}{2}(\rho - 2)$       E) NOTA

25. Let  $n$  be an integer greater than 1. Evaluate  $\int x^{n-1} \ln x \, dx$ .

- A)  $nx^{n-1} + C$       B)  $nx^n - \ln x + C$       C)  $\frac{1}{2}x^n \ln^2 x + C$       D)  $x^{n-2} - x \ln x + C$       E) NOTA

26. The region between the curves  $x = \sqrt{y}$  and  $x = y^2$  in the first quadrant is the base of a solid. The cross sections perpendicular to the  $y$ -axis are rectangles whose height is  $420\rho \ln 5$  times the length of the plane of the base. What is the volume of the solid?

- A)  $54\rho \ln 5$       B)  $63\rho \ln 5$       C)  $72\rho \ln 5$       D)  $140\rho \ln 5$       E) NOTA

27. Let  $a$  be a positive real number. Given that

$$\int_{-a}^{2a} (x^3 - (a+1)x^2 - 3ax + 4a) dx = 0,$$

compute the sum of the possible values of  $a$ .

- A) 6      B) 8      C) 10      D) 12      E) NOTA

28.  $\int_0^\rho \int_x^\rho \frac{\sin y}{y} dy dx =$

- A) 1      B) 2      C)  $\rho$       D)  $\rho^2$       E) NOTA

29. Evaluate  $\int_{\pi/6}^{\pi/4} (\sec x \tan^2 x + \sec^3 x - \tan x - \sec^2 x \cot x) dx$ .

- A)  $\sqrt{2} - \sqrt{3} - 1 + \ln|5\sqrt{2} - \sqrt{3} - 2|$   
 B)  $\sqrt{2} + 1 - \sqrt{3} - \ln|\sqrt{6} + \sqrt{3}|$   
 C)  $\sqrt{2} - 2\sqrt{3} - \frac{1}{2} \ln 24$   
 D)  $\sqrt{2} - \frac{2}{3} + \ln \frac{\sqrt{2}}{3}$   
 E) NOTA

30.  $\int_0^\forall e^{-x^2} dx =$

- A) 1      B)  $\sqrt{\rho}$       C)  $\frac{5}{2} \ln 2$       D)  $\forall$       E) NOTA