For all questions, answer "(E) NOTA" means none of the above answers is correct. All numbers on this test are real numbers. All functions on this test have domains and ranges that are subsets of the real numbers.

1. Evaluate: $\lim_{x\to\infty}\left(\frac{5x+3}{4x-x^2}+\frac{9x^3-4x^2+7x-100}{3x^3+2x^5-20x+1}\right)$

(A) 16/3 (B) 17/4 (C) 18/5 (D) 19/6 (E) NOTA

2. If $f(x) = x\sin(x)$, find the value of $f'(2015\pi)$.

(A) -2015π (B) π (C) 2015π (D) 0 (E) NOTA

3. Evaluate: $\lim_{x\to 1^-}\left(\frac{5x-3}{2x+1}\right)$

(A) 2/5 (B) 2/3 (C) 3/2 (D) 5/2 (E) NOTA

4. If $P(x) = 5x^3 - 100x^2 + 17x^5 - 10^{10^{10^{10^{10}}}}x$, what is $\frac{d^2P}{dx^2}$?

(A) $\frac{1}{4}x^5 - \frac{25}{3}x^4 + \frac{17}{42}x^7$ (B) $15x^2 - 200x + 85x$

(C) $30x^2 - 200x + 170x^2$ (D) $340x^3 + 30x - 200$ (E) NOTA

5. Denote the greatest integer less than or equal to N as $\lfloor N\rfloor$. Let $f(x) = \lfloor 10x^2 - 3x - 7\rfloor$. Evaluate: $\left(\lim_{x\to -.50^+} f(-x)\right) + \left(\lim_{x\to .50^-} f(x)\right)$

(A) -15 (B) -14 (C) -13 (D) -12 (E) NOTA

6. Find the smallest integer greater than $\lim_{h\to 0}\frac{\left(3(2+h)^2-\sqrt{h+9}+24+12h\right)-33}{h}$.

(A) 25 (B) 24 (C) 23 (D) 22 (E) NOTA

7. Evaluate: $\left\lfloor \lim_{n\to\infty}\sum_{i=1}^{n}\left(\left(\frac{5n+3i}{n}\right)^4\frac{1}{n}\right)\right\rfloor$

(A) 1986 (B) 1981 (C) 1976 (D) 1971 (E) NOTA

8. If $F(x) = \int_{x^{1/3}}^{2015} 12\sin(t^6)\,dt$, find $F'(x)$.

(A) $12\sin x^2$ (B) $-4\sin x^2$

(C) $\frac{12\cos x^2}{2015x^{1/3}}$ (D) $\frac{-4\sin x^2}{x^{2/3}}$ (E) NOTA

9. Triangle ABC is a right triangle, with right angle ∠ACB and AC = 1. Let D and E be points on $\overline{\text{AB}}$ and $\overline{\text{BC}}$, respectively, such that AD = AC and $m\angle\text{CDE} = m\angle\text{CAB} = \theta$, where θ is in radians. The line perpendicular to $\overline{\text{BC}}$ passing through E intersects $\overline{\text{AB}}$ at point F. Let $f(\theta)$ denote the length of $\overline{\text{EF}}$. If $\lim_{\theta\to 0^+} f(\theta) = x/y$, where x and y are relatively prime positive integers, find $x + y$.

(A) 3 (B) 4 (C) 5 (D) 6 (E) NOTA

10. Find the average rate of change of $f(x) = (2x+4)^3$ on the interval $-4 \le x \le 1$.

(A) 26 (B) 36 (C) 46 (D) 56 (E) NOTA

11. The graph of the cubic $y = x^3 - 9x^2 + 24x + 4$ has a local minimum at (x_1, y_1) and a local maximum at (X_2, Y_2). Find the determinant of the matrix $\begin{pmatrix} x_1 & X_2 \\ y_1 & Y_2 \end{pmatrix}$.

(A) 56 (B) -56 (C) 136 (D) -136 (E) NOTA

12. Which of the following is necessarily true regarding the graph of $y = f(x) = 5\sin x + 12\cos^2 x$ at the point $P = \big(2\pi, f(2\pi)\big)$?

(A) The graph is concave up at P and $f'(2\pi) > 0$.

(B) The graph is concave down at P and $f'(2\pi) < 0$.

(C) The graph is concave up at P and $f'(2\pi) < 0$.

(D) The graph is concave down at P and $f'(2\pi) > 0$.

(E) NOTA

13. Which of the following is the best reason for why the Mean Value Theorem for Derivatives does not necessarily hold for the function

$$f(x) = 2|x-5| + \sin(x^2 - 10x + 25)$$

on the interval $x \in [-10,10]$?

(A) The value of $f(-10)$ does not equal the value of $f(10)$.

(B) The function f is not differentiable for all x on the interval $(-10,10)$.

(C) The period of f is not an integer.

(D) The Mean Value Theorem for Derivatives does not apply to trigonometric functions.

(E) NOTA

14. Function f is defined for real numbers as $f(x) = nx^3 - x$ if $x \leq 1$ and $f(x) = 5 + mx^2$ if $x > 1$, where m and n are constants with respect to x and f. If f is differentiable for all x, find the value of $|m| + |n|$.

(A) 24 (B) 25 (C) 26 (D) 27 (E) NOTA

15. Suppose that $L(x) = ax + b$ is the equation of the line normal to the graph of $y = 8(3x+4)^{-1/2}$ at the point $\left(4, L(4)\right)$. The x-intercept of the graph of $y = L(x)$ is $(m/n, 0)$, where m and n are relatively prime positive integers. Find the value of $m + n$.

(A) 9 (B) 37 (C) 47 (D) 73 (E) NOTA

16. Denote $F(x)$ as the derivative of $x^{5/3}$ with respect to $x^{-7/2}$. The value of $|F(1)|$ is equal to m/n, where m and n are relatively prime positive integers. Find $m + n$.

(A) 17 (B) 23 (C) 31 (D) 37 (E) NOTA

17. If $P(x) = (2015x - 1)^{100}$, find the constant term of $P'(x)$ when expanded and like-terms are combined.

(A) -9900 (B) 201500 (C) 9900 (D) -201500 (E) NOTA

18. The function $y(x)$, where $y(0) = 0$, satisfies the differential equation $\frac{dy}{dx} = 2e^{-2y}$. Find the value of $y(2070)$.

(A) $\ln 91$ (B) $\ln 101$ (C) $\ln 111$ (D) $\ln 121$ (E) NOTA

19. If $f(x) = 3x^2 - x$ for $x > 1$ and $g(x) = f^{-1}(x)$, the value of $g'(10) = m/n$, where m and n are relatively prime positive integers. Find the value of $m + n$.

(A) 10 (B) 11 (C) 12 (D) 13 (E) NOTA

20. Let O = (0,0) be the origin and P $= (p, p^2)$ a point on the parabola $y = x^2$, where $p \neq 0$. Let Q be a point on the parabola distinct from P and O such that $\overleftrightarrow{\mathrm{QO}}$ is perpendicular to $\overleftrightarrow{\mathrm{PO}}$. The smallest possible area of triangle POQ is equal to m/n, where m and n are relatively prime positive integers. Find $m + n$.

(A) 5 (B) 6 (C) 7 (D) 8 (E) NOTA

21. Evaluate: $\lim_{h\to 0}\left(\frac{1}{h}\left(-1 + \sum_{n=0}^{\infty}\left(\frac{(-1)^n h^{2n+1}}{(2n+1)!} + \frac{h^n}{n!}\right)\right)\right)$

(A) 3 (B) 2 (C) 1 (D) 0 (E) NOTA

22. If $f(x) = -100 + \sum_{k=1}^{100} x^k$, find the largest positive prime divisor of $f'(1)$.

(A) 83 (B) 89 (C) 97 (D) 101 (E) NOTA

23. If $y = \sin^4 x + \cos^4 x$ and $\mathrm{F}(x) = \frac{d^{2015}y}{dx^{2015}}$, find the value of $\log_2\left(\mathrm{F}\left(\frac{\pi}{16}\right)\right)$.

(A) 2013.50 (B) 2014.50 (C) 4027.50 (D) 4029.50 (E) NOTA

24. Let $f(x) = x^3 - 2$. Using an initial value of $r_0 = 1$, applying two iterations of the Newton-Raphson method to approximate the real root of f yields a value of r_2. Given that $r_2 = m/n$, where m and n are relatively prime positive integers, find $m - n$.

(A) 7 (B) 19 (C) 31 (D) 163 (E) NOTA

25. Let $L = \lim_{x\to\infty}\left(\sqrt{x^2 + 4x + 17} - \sqrt[3]{x^3 - 12x^2 + 48x - 20}\right)$

and for $f(x) = \frac{(x^2+6)(3x^2-2x+6)+(2x)(6-6x+x^2-x^3)}{(x^2+6)^2}$, let $D = f(2013)$. Find the product LD.

(A) 6 (B) 4 (C) 2 (D) 0 (E) NOTA

26. The equation $e^{2x} = k\sqrt{x}$ has exactly one real solution x for some constant k. Find the greatest integer less than or equal to $100k^2$.

(A) 271 (B) 738 (C) 1087 (D) 2446 (E) NOTA

27. For positive real numbers m and n, where $m < n$, let $R(m,n)$ equal the area of region in the first quadrant of the Cartesian Plane bounded by the graphs of $y = x^{m/n}$ and $y = x^{n/m}$. If $\frac{dm}{dt} = -\frac{9}{2}$ and $\frac{dn}{dt} = \frac{11}{2}$, find the value of $\frac{dR}{dt}$ when $m = \frac{9}{2}$ and $n = \frac{11}{2}$.

(A) $\frac{1}{10}$ (B) $\frac{27}{100}$ (C) $\frac{63}{100}$ (D) $\frac{99}{100}$ (E) NOTA

28. Evaluate: $\lim_{n\to\infty}\left(\frac{n+4}{n+8}\right)^{2n}$

(A) 0 (B) $\frac{1}{e^{16}}$ (C) e^{-8} (D) 1 (E) NOTA

29. Let $F(x) = \int_x^{4x} \cos\left(\frac{1}{t}\right) dt$ and $L = \lim_{x\to\infty}\frac{F(x)}{x}$. Find the value of L.

(A) 4 (B) 3 (C) 2 (D) 1 (E) NOTA

30. Let f be a function where f''' is continuous and all of $f(x)$, $f'(x)$, $f''(x)$, and $f'''(x)$ are positive for all x. Also, suppose that $f'''(x) \leq f(x)$ for all x. Let $F(x) = \ln(f(x))$. Which of the following is a possible value of $F'(0)$?

(A) e (B) 9001 (C) π (D) e^{2015} (E) NOTA