1. E (0)
2. A
3. B
4. D
5. B
6. B
7. C
8. D
9. B
10. D
11. A
12. D
13. B
14. E (28)
15. B
16. C
17. D
18. A
19. C
20. E
21. B
22. D
23. C
24. B
25. A
26. C
27. D
28. C
29. B
30. E (Has to be less than 2.)

1. E. Notice that in each fraction, the degree of the denominator is greater than the degree of the numerator. Thus, the limit equals $\mathbf{0}$.
2. A. By the Product Rule, $f'(x) = \sin x + x\cos x$, so $f'(2015\pi) = \boldsymbol{-2015\pi}$.
3. B. Direct Substitution easily yields $\mathbf{2/3}$ as the limit.
4. D. Ignore the linear term with the super large coefficient; it will go away upon second-differentiation. Using the Power Rule twice yields $\mathbf{340x^3 + 30x - 200}$.
5. B. Careful! We have $(\lim_{x\to -.50^+} f(-x)) + (\lim_{x\to .50^-} f(x)) = -7 + (-7) = \mathbf{-14}$.
6. B. The limit can be interpreted as a derivative, but using L'Hopital's Rule is probably easier. The limit is equal to

$$\lim_{h\to 0}\left(6(2+h) - \frac{1}{2\sqrt{h+9}} + 12\right) = 12 - \frac{1}{6} + 12 = 23\frac{5}{6}.$$

   The smallest integer greater than this number is $\mathbf{24}$.
7. C. Rather than using a bunch of crazy summation formulae for sum of powers of integers, the limit can be interpreted as an integral. Namely,

$$\int_0^1 (5+3x)^4\,dx = 1976.2,$$

   so the answer is **1976**.
8. D. Using the Fundamental Theorem of Calculus and the Chain Rule yields

$$F'(x) = -12\sin\left(x^{\frac{1}{3}}\right)^6 \cdot \left(\frac{1}{3}x^{-\frac{2}{3}}\right) = \frac{\mathbf{-4\sin x^2}}{\mathbf{x^{2/3}}}.$$

9. B. All arguments are in degrees. Notice that triangle BFE is similar to triangle BAC, so that $m\angle BFE = m\angle BAC = \theta$. Also, $\frac{FE}{AC} = \frac{BE}{BC}$, or $EF = \frac{BE}{BC} = \frac{BE}{BD}\cdot\frac{BD}{BC}$. Now, let's do some angle chasing. Since AD = AC, this means triangle ADC is isosceles and $m\angle ADC = m\angle ACD = 90 - \theta/2$. Also, $m\angle BCD = 90 - m\angle ACD = \theta/2$, $m\angle DEC = 180 - m\angle CDE - m\angle BCD = 180 - 3\theta/2$, $m\angle BDE = 180 - m\angle ADC - m\angle CDE = 90 - \theta/2$, and finally, $m\angle BED = 90 + m\angle FED = 3\theta/2$. Using the Law of Sines on triangle BED and BDC, yields the equations

$$\frac{BE}{BD} = \frac{\sin\left(90-\frac{\theta}{2}\right)}{\sin\left(\frac{3\theta}{2}\right)} = \frac{\cos\left(\frac{\theta}{2}\right)}{\sin\left(\frac{3\theta}{2}\right)} \quad\text{and}\quad \frac{BD}{BC} = \frac{\sin\left(\frac{\theta}{2}\right)}{\sin\left(\frac{\theta}{2}+90\right)} = \frac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)}.$$

   Thus, $f(\theta) = EF = \frac{BE}{BD}\cdot\frac{BD}{BC} = \frac{\cos\left(\frac{\theta}{2}\right)}{\sin\left(\frac{3\theta}{2}\right)}\cdot\frac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)} = \frac{\sin\left(\frac{\theta}{2}\right)}{\sin\left(\frac{3\theta}{2}\right)}$.

   As θ approaches 0 from the positive side, this expression approaches 1/3. Thus, $m + n = 1 + 3 = \mathbf{4}$.

10. D. The average rate of change is given by

$$\frac{f(1)-f(-4)}{1-(-4)} = \frac{1}{5}\big(f(1)-f(-4)\big) = \frac{1}{5}\big(216-(-64)\big) = \mathbf{56}.$$

11. A. The critical values are the solutions to $y' = 3x^2 - 18x + 24 = 0$, or $x \in \{2,4\}$. By the First Derivative Test, the point $\big(4, y(4)\big) = (4,20) = (x_1, y_1)$ is a local minimum while $\big(2, y(2)\big) = (2,24) = (X_2, Y_2)$ is a local maximum. The desired matrix is then equal to $\begin{pmatrix} 4 & 2 \\ 20 & 24 \end{pmatrix}$, which has a determinant of $(4)(24) - (2)(20) = \mathbf{56}$.

12. D. Since $f'(P) > 0$ and $f''(P) < 0$, Answer Choice **D** follows.

13. B. The Mean Value Theorem requires that the function be differentiable on the open interval in order to guarantee a solution. Answer Choice **B** is the answer.

14. E. Differentiability implies continuity. Thus, $f(1^-) = f(1^+)$ and $f'(1^-) = f'(1^+)$. The first equation yields $n - 1 = 5 + m$ while the second equation yields $3n - 1 = 2m$. Solving this system yields $(m,n) = (-17,-11)$, so $|m| + |n| = 17 + 11 = \mathbf{28}$.

15. B. We have $L(4) = y(4) = 2$. We have

$$y'(x) = \left(-\frac{1}{2}\right)8(3x+4)^{-\frac{3}{2}}(3) = -12(3x+4)^{-\frac{3}{2}},$$

so $y'(4) = -\frac{3}{16}$, giving a perpendicular slope of $\frac{16}{3}$. Therefore, the x-intercept of the line is the solution to the equation $0 - 2 = \left(\frac{16}{3}\right)(x-4)$, or $x = \frac{29}{8} = \frac{m}{n}$, so $m + n = 29 + 8 = \mathbf{37}$.

16. C. We have $F(x) = \frac{\left(\frac{5}{3}\right)x^{\frac{2}{3}}}{-\left(\frac{7}{2}\right)x^{-\frac{9}{2}}}$, so that $|F(1)| = \left|-\frac{10}{21}\right| = \frac{10}{21}$, so $m + n = 10 + 21 = \mathbf{31}$.

17. D. The constant term of a polynomial is just the polynomial evaluated at 0. Since $P'(x) = 100(2015x-1)^{99}(2015) = 201500(2015x-1)^{99}$, the answer is just $P'(0) = 201500(-1)^{99} = \mathbf{-201500}$.

18. A. By Separation of Variables, we have $\frac{1}{2}e^{2y} = 2x + C$, and using the initial conditions, we find that $C = \frac{1}{2}$. Thus, we find that $y(x) = \ln\sqrt{4x+1}$, making $y(2070) = \mathbf{\ln 91}$.

19. C. Note that $f'(x) = 6x - 1$. We have $f(2) = 10$, so that means that $g(10) = 2$ and $g'(10) = \frac{1}{f'(2)} = \frac{1}{11}$, so $m + n = 1 + 11 = \mathbf{12}$.

20. E. The slope connecting segment PO is $\frac{p^2-0}{p-0} = p$; therefore, the slope of line QO is $-\frac{1}{p}$, making the equation of line QO to be $y = -\frac{1}{p}x$. This line intersects the parabola when $-\frac{1}{p}x = x^2$, or $x = -\frac{1}{p}$. Therefore, Q $= \left(-\frac{1}{p}, \frac{1}{p^2}\right)$. By the determinant formula for the area of a triangle, the area of triangle POQ is the absolute value of

$$\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ p & p^2 & 1 \\ -1/p & 1/p^2 & 1 \end{vmatrix} = \frac{p^2+1}{2p}.$$

This expression will attain its minimum when $p = 1$, yielding a minimum area of $1 = \frac{1}{1}$, so $m + n = 1 + 1 = \mathbf{2}$.

21. B. Converting the Power Series to explicit form, we find that the limit is equal to $\lim_{h\to 0} \frac{1}{h}(-1 + \sin h + e^h)$. By L'Hopital's Rule, we have $\lim_{h\to 0}(\cos h + e^h) = 1 + 1 = \mathbf{2}$.

22. D. By the Power Rule,

$f'(x) = \sum_{k=1}^{100} kx^{k-1}$, so $f'(1) = \sum_{k=1}^{100} k = \frac{(100)(101)}{2} = 5050 = 2 \times 5^2 \times 101$.

The largest positive prime divisor is **101**.

23. C. Notice that

$$y = \sin^4 x + \cos^4 x = (\sin^2 x + \cos^2 x)^2 - 2(\sin x \cos x)^2 = 1 - 2\left(\frac{1}{2}\sin(2x)\right)^2$$
$$= \frac{3}{4} + \frac{1}{4}\cos(4x).$$

Thus, the trigonometric term in the derivatives of y are going to cycle with a period of 4, with the coefficient getting multiplied by 4 with each differentiation. Since $2015 \equiv 3$ in modulo 4, the trigonometric term of $y^{(2015)}$ is essentially the trigonometric term of $(\cos(4x))'''$, or $\sin(4x)$. Taking derivatives 2015 times yields 2015 4's, so $F(x) = \left(\frac{1}{4}\sin(4x)\right)(4^{2015}) = 4^{2014}\sin(4x) = 2^{4028}\sin(4x)$. Thus, $F\left(\frac{\pi}{16}\right) = 2^{4028}\sin\left(\frac{\pi}{4}\right) = 2^{4028-\frac{1}{2}} = \mathbf{2^{4027.50}}$.

24. B. By Newton's Method, we have

$r_{n+1} = r_n - \frac{f(r_n)}{f'(r_n)} = r_n - \frac{r_n^3-2}{3r_n^2}$, so that $r_1 = 1 - \frac{1^3-2}{3(1)^2} = \frac{4}{3}$, and $r_2 = \frac{4}{3} - \frac{\left(\frac{4}{3}\right)^3-2}{3\left(\frac{4}{3}\right)^2} = \frac{91}{72}$,

so $m - n = \mathbf{19}$.

25. A. The trick to quickly evaluating L is to add (or subtract) just the right constants under the radicals so that the polynomials factor nicely while not changing the value of the limit. In this case, subtracting 13 under the square root and subtracting 44 under the cube root yields

$$L = \lim_{x\to\infty}\left(\sqrt{x^2+4x+4} - \sqrt[3]{x^3-12x^2+48x-64}\right)$$

$$L = \lim_{x\to\infty}\left(\sqrt{(x+2)^2} - \sqrt[3]{(x-4)^3}\right)$$

$$L = \lim_{x\to\infty}\left((x+2)-(x-4)\right) = 6.$$

Now, the trick to quickly evaluate D is to interpret f as the derivative of some quotient, namely, $\frac{x^3-x^2+6x-6}{x^2+6} = x-1$. Thus, $f(x) = (x-1)' = 1$ for all values of x. Specifically, $D = f(2013) = 1$, making $LD = (6)(1) = \mathbf{6}$.

26. C. First, note that k has to be positive; otherwise, the graphs of the respective expressions will not even have a prayer of intersecting, as the exponential in the equation is always positive. Also, since the exponential graph is concave up and the square root graph is concave down, the only way they will intersect exactly once is if the two graphs are tangent to each other. Thus, $(e^{2x})' = \left(k\sqrt{x}\right)'$, or $2e^{2x} = \frac{k}{2\sqrt{x}}$. Combining this with the original equation, $e^{2x} = k\sqrt{x}$, we get $x = \frac{1}{4}$, so that $k = 2\sqrt{e}$, or $100k^2 = 400e \approx 4(2.718) \approx \mathbf{1087}$.

27. D. Let $A = n-m$ and $B = n+m$. By the linearity of differentiation, $A' = n'-m'$ and $B' = n'+m'$. Note that for the values given in the problem, $A = 1$, $B = 10$, $A' = 10$, and $B' = 1$. Now, on to the problem at hand. The two graphs intersect at $x = 0$ and $x = 1$. By integration, the area is

$$R(m,n) = \int_0^1 \left(x^{m/n} - x^{n/m}\right) dx = \frac{n-m}{n+m} = \frac{A}{B}.$$

By the Quotient Rule, $\frac{dR}{dt} = \frac{A'B-AB\prime}{B^2}$, which, after plugging in the respective values, yields $\frac{(10)(10)-(1)(1)}{10^2} = \frac{\mathbf{99}}{\mathbf{100}}$.

28. C. The limit can be written as $\lim_{n\to\infty}\frac{\left(1+\frac{4}{n}\right)^{2n}}{\left(1+\frac{8}{n}\right)^{2n}} = \frac{e^{4\times 2}}{e^{8\times 2}} = e^{8-16} = \boldsymbol{e^{-8}}$.

29. B. By the Mean Value Theorem for Integrals, there exists a $c \in (x, 4x)$ such that $\frac{\int_x^{4x}\cos\left(\frac{1}{t}\right)dt}{3x} = \cos\left(\frac{1}{c}\right)$. Now, as x approaches infinity, c also grows without bound, so that $\lim_{x\to\infty}\frac{F(x)}{x} = \lim_{c\to\infty} 3\cos\left(\frac{1}{c}\right) = \mathbf{3}$.

30. E. We will show that $F'(0) < 2$, disqualifying all answer choices A through D. Since $F'(0) = \frac{f'(0)}{f(0)}$, we'll be done if we can show that $f'(0) < 2f(0)$. Note that since f' is positive, the function f is strictly increasing. Thus, if $x \leq 0$, we have $f(x) \leq f(0)$ and combining this with the hypotheses of the problem yields $f'''(x) \leq f(x) \leq f(0)$, or $f'''(x) \leq f(0)$. Integrating both sides twice—and minding the constants of integration in each step—yields $f'(x) \leq f'(0) + xf''(0) + \frac{1}{2}x^2 f(0)$. The coefficients of the right-hand side of this inequality are positive and since $f'(x)$ is also positive, this means that the quadratic given by $P(x) = f'(0) + xf''(0) + \frac{1}{2}x^2 f(0)$ has no nonnegative zeroes. Therefore, the discriminant of P must be negative, or $\left(f''(0)\right)^2 - 4\left(\frac{1}{2}f(0)\right)\left(f'(0)\right) = \left(f''(0)\right)^2 - 2f'(0)f(0) < 0$. We can proceed using similar reasoning to the relationship between f''' and f'' to find that $\left(f'(0)\right)^2 - 2f(0)f''(0) < 0$. Combining these two inequalities yields $\left(f'(0)\right)^4 < 4\left(f(0)\right)^2\left(f''(0)\right)^2 < 8\left(f(0)\right)^3 f'(0)$, which implies that $f'(0) < 2f(0)$, and we're done.