For each question, "E) NOTA" indicates that none of the above answers is correct.

1. Let $y = \sqrt{x + \sqrt{x + \sqrt{x + \cdots}}}$. Find $\frac{dy}{dx}$.

A) $\frac{1}{2x-1}$ B) $\frac{1}{2y-1}$ C) $2y - 1$ D) $\frac{x}{2y-1}$ E) NOTA

2. $y = \arctan\frac{2}{x}$. Find $\frac{dy}{dx}$.

A) $-\frac{2}{x^2+4}$ B) $\frac{2}{x^2+4}$ C) $-\frac{2}{\sqrt{4-x^2}}$ D) $\frac{2}{\sqrt{4-x^2}}$ E) NOTA

3. If the space diagonal of a cube is increasing at a rate of 3 inches per minute, how fast is the side of the cube increasing, in inches per minute?

A) $\frac{\sqrt{2}}{2}$ B) $\frac{3\sqrt{2}}{2}$ C) $\sqrt{3}$ D) $\frac{\sqrt{3}}{3}$ E) NOTA

4. Suppose the function $g(x) = f(x) - f(2x)$ has derivative 5 at $x = 1$ and derivative 7 at $x = 2$. Find the derivative of $h(x) = f(x) - f(4x)$ at $x = 1$.

A) -9 B) 0 C) 12 D) 19 E) NOTA

5. Calculate the maximum value of

$$\sum_{k=1}^{n}\left(\frac{(-1)^k}{k!}\int_0^k 3x^2dx\right)$$

A) 2 B) 3 C) 4 D) $2\sqrt{e}$ E) NOTA

6. Evaluate: $\lim_{x\to\infty}\left(1 - \frac{3}{2x}\right)^{2x}$

A) DNE B) e^{-3} C) e^{-2} D) e E) NOTA

7. Let the particular solution to the differential equation: $x\frac{dy}{dx} + y = x$ be $y = f(x)$ with the initial condition $f(1) = 1$. What is $f(2)$? (Hint: If stuck, think about derivative rules).

A) 0 B) 16 C) 32 D) 64 E) NOTA

8. Let $x^2 + y^2 = 9$. Find $\frac{d^2y}{dx^2}$.

A) $-\frac{9}{x^3}$ B) $-\frac{9}{y^3}$ C) $\frac{3}{y^3}$ D) $\frac{3}{x^3}$ E) NOTA

9. Evaluate the following limit:

$$\lim_{x\to 0} \frac{-1-x+e^{xcos(x)}}{\sin(x^2)}$$

A) DNE B) 0 C) $\frac{1}{2}$ D) $\frac{3}{2}$ E) NOTA

10. Let f be a function such that $f(0) = 1$, $f'(0) = 2$ and $f''(t) = 4f'(t) - 3f(t) + 1$ for all t. Compute the 4th derivative of f evaluated at $x = 0$.

A) 0 B) 54 C) 64 D) 72 E) NOTA

11. Let a_n be the value of c guaranteed by the Mean Value Theorem for derivatives over the interval [0, $\left(\frac{1}{2}\right)^n$] for the function $f(x) = x^3$. Find:

$$\sum_{n=1}^{\infty} a_n$$

A) 1 B) $\sqrt{3}$ C) $\frac{\sqrt{3}}{3}$ D) $\frac{2\sqrt{3}}{3}$ E) NOTA

12. Evaluate the limit: $\lim_{x\to 2}(\frac{1}{x-2} - \frac{4}{x^2-4})$

A) DNE B) 0 C) $\frac{1}{2}$ D) $\frac{1}{4}$ E) NOTA

13. Let $f(x) = x^3 + ax + b$, with $a \neq b$, and suppose the tangent lines to the graph of f at $x = a$ and $x = b$ are parallel. Find $f(1)$.

A) 1 B) 3 C) $2b + 1$ D) $2a + 1$ E) NOTA

14. Evaluate the following limit:

$$\lim_{x\to -\infty} \frac{4x-1}{\sqrt{x^2+2}}$$

A) DNE B) 0 C) 2 D) 4 E) NOTA

15. Find the value $a > 0$ that maximizes the product of the roots of $f(x) = 2x^2 + a^2 + ax - 1$ given that $f(x)$ has only real roots.

A) Any value of a B) $-\frac{1}{2}$ C) $\frac{2\sqrt{2}}{7}$ D) $\frac{2\sqrt{7}}{7}$ E) NOTA

16. If $f(x) = x \ln x - x$, evaluate $\lim_{h\to 0} \frac{f(x+2h)-f(x-2h)}{2h}$ when $x = 4$.

A) $\ln 2$ B) $2\ln 2$ C) $3 \ln 2$ D) $4 \ln 2$ E) NOTA

17. A radioactive substance decays exponentially. What is the average quantity present over the first half-life if a sample begins with 10 grams?

A) 7.5 B) $\frac{10}{\ln 2}$ C) $\frac{5}{\ln 4}$ D) $\frac{5}{\ln 2}$ E) NOTA

18. Find the length of $r = \theta^2$ from $\theta = 0$ to $\theta = \sqrt{5}$.

A) $\frac{65}{4}$ B) $\frac{19}{3}$ C) $\frac{125}{3}$ D) 7 E) NOTA

19. Evaluate the following limit:

$$\lim_{x\to 0} \left(\frac{\sin x}{x}\right)^{\frac{1}{1-\cos x}}$$

A) 1 B) e C) $e^{1/3}$ D) $e^{2/3}$ E) NOTA

20. A sample of gas obeys Boyle's law, $PV = k$, where k is a constant. It occupies a volume V of 1000 cubic inches at $P = 10$ pounds per square inch, and the gas is compressed at a rate of 12 cubic inches per minute. Find the rate the pressure increases, in pounds per square inch per minute, at the instant when the volume is equal to 600 cubic inches.

A) 1/10 B) 1/3 C) 3/20 D) 1/5 E) NOTA

21. If $f(x) = \frac{1}{x^2+x+1}$, find $f^{(36)}(0)$.

A) $\frac{1}{36!}$ B) $\frac{1}{72!}$ C) $36!$ D) $72!$ E) NOTA

22. Find a function $y^{(n)}$ for all derivatives after the 1st derivative (i.e. when n=2,3,4…) of the function $y = \sqrt{2x-1}$.

A) $\frac{(-1)^n(2n-2)!}{(\sqrt{2x-1})^{2n-1}}$ B) $\frac{(-1)^n(2n-3)!}{(\sqrt{2x-1})^{2n}}$ C) $\frac{(-1)^{n+1}(2n-3)!!}{(\sqrt{2x-1})^{2n-1}}$ D) $\frac{(-1)^{n+1}(2n-2)!}{(2x-1)^{2n-1}}$ E) NOTA

23. What is the minimum vertical distance between the graphs of $2 + \sin x$ and $\cos x$?

A) $2 - \sqrt{2}$ B) $\sqrt{2}$ C) $\sqrt{2} + \frac{\sqrt{2}}{2}$ D) $\sqrt{2} + 2$ E) NOTA

24. Suppose that $f(x)$ is a polynomial such that $f(x) - f'(x) = x^2 + 2x + 1$. Compute $f(5)$.

A) 38 B) 48 C) 50 D) 53 E) NOTA

25. Find the n^{th} nonzero term of the Maclaurin series for $\ln(1 + x^2)$.

A) $\frac{(-1)^n x^{2n}}{2n}$ B) $\frac{(-1)^{n+1} x^{2n-1}}{2n}$ C) $\frac{(-1)^{n+1} x^{2n-1}}{n}$ D) $\frac{(-1)^n x^{2n}}{n}$ E) NOTA

26. A nonzero polynomial $f(x)$ with real coefficients has a property that $f(x) = f'(x) * f''(x)$. What is the leading coefficient of $f(x)$?

A) $\frac{1}{18}$ B) $\frac{1}{9}$ C) $\frac{1}{6}$ D) $\frac{1}{3}$ E) NOTA

27. Evaluate the following limit:

$$\lim_{x \to 1} x^{\frac{x}{\sin(1-x)}}$$

A) e^{-2} B) e^{-1} C) 1 D) e E) NOTA

28. What is the radius of an expanding circle at a moment when the rate of change of its enclosed area is numerically twice as large as the rate of change of its radius?

A) $\frac{1}{\pi}$ B) $\frac{2}{\pi}$ C) π D) 2π E) NOTA

29. Let $f(x) = \int_{0.1}^{2x} (\cot t + 1)\, dt$. Find $f'(\frac{\pi}{6})$.

A) $\frac{\sqrt{3}}{3}$ B) $\frac{2\sqrt{3}}{2} + 2$ C) $\frac{2\sqrt{3}}{3}$ D) $\frac{2\sqrt{3}}{3} + 2$ E) NOTA

30. The following curve is defined by the parametric equations: $x = 2\sin\theta$ and $y = \cos 2\theta$. What is $\frac{d^2y}{dx^2}$?

A) $-2\cos\theta$ B) $-2\sin\theta$ C) -1 D) -2 E) NOTA