You have 60 minutes to complete this 30-question test. Do not get bogged down by any one question; some are significantly more difficult than the others. The answer choice E. NOTA denotes that "None of These Answers" is correct. Good luck!

For each of the following functions in **questions 1-3**, classify the behavior at the point **(x,y) = (0,0)**. Choose from one of the following: NOT (not a critical point), MAX (local maximum), MIN (local minimum), SAD (saddle point), and NOTA (none of the other answers).

1. $f(x,y) = x + x^2 + y^2$
A) NOT B) MAX C) MIN D) SAD E) NOTA

2. $f(x,y) = x^3 + 2x^2y^2 + y^3$
A) NOT B) MAX C) MIN D) SAD E) NOTA

3. $f(x,y) = x^2 + 2xy + y^2$
A) NOT B) MAX C) MIN D) SAD E) NOTA

4. Compute the integral: $\int_0^{\pi/2}\int_x^{\pi/2} \frac{\sin y}{y}\, dydx.$
A) -1 B) 1 C) ½ D) -½ E) NOTA

For **questions 5 and 6**, consider the following points in R^3.
A = (0, -1, 2) , B = (1, -1, 1) , C = (5, 2, 0) , D = (-1, 4, -1) .

5. Find an equation for the plane through A, B and C. When written in the form x + ay + bz = c, what is the value of a + b + c?
A) -1 B) 1 C) 3 D) 5 E) NOTA

6. Find an equation for the line which goes through D and intersects the plane in #5 orthogonally.
A) s(t) = (t-1, 4-t, t-1) B) s(t) = (t-1, 4+t, t+1) C) s(t) = (t, t, t)
D) s(t) = (t+1, t+4, t+1) E) NOTA

7. Which of the following equalities is neither derived from nor is a form of Green's Theorem?
A) $\int_C Pdx + Qdy = \int_D (\frac{\partial Q}{\partial x} + \frac{\partial P}{\partial y})dxdy$ B) $\int_{\partial D} \mathbf{F} \cdot d\boldsymbol{s} = \int_D (\text{curl } \mathbf{F}) \cdot \mathbf{k} dA$
C) $\int_{\partial D} \mathbf{F} \cdot \mathbf{n} ds = \int_D \text{div } \mathbf{F}\, \boldsymbol{dA}$ D) Area $= \frac{1}{2}\int_{\partial D} xdy - ydx$ E) NOTA

For **questions 8-10**, consider the parameterized curve in R^3 given by
$$\vec{r}(t) = \left(\frac{t^3}{3} - t, t^2 - 1, 0\right)$$
for $-2 \le t \le 2$.

8. Find the acceleration of a particle moving along the curve at time t = 1.
A) (0, 2, 0) B) (2, 2, 0) C) (2, 0, 0) D) (2, 2, 2) E) NOTA

9. Find the speed of a particle moving along the curve at time t = 2.
A) -5 B) 2 C) 3 D) 4 E) NOTA

10. Find the arc length of the curve for $-2 \le t \le 2$.
A) 5 B) 14/3 C) 20/3 D) 28/3 E) NOTA

For **questions 11 and 12**, determine the value of the following limits, if they exist.

11. $\lim_{(x,y)\to(0,0)} \frac{x^3y^3}{x^4+y^2}$.
A) DNE B) -1 C) 0 D) 1 E) NOTA

12. $\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$.
A) DNE B) -1 C) 0 D) 1 E) NOTA

13. Let **F** be a vector field continuous on the C^1 path $\boldsymbol{\sigma}$: $[a_1, b_1] \to R^3$. Let $\boldsymbol{\rho}$: $[a, b] \to R^3$ be a reparametrization of $\boldsymbol{\sigma}$. If $\boldsymbol{\rho}$ is orientation-reversing, and the value of $\int_\sigma \boldsymbol{F}\cdot d\boldsymbol{s} = 13$, what is the value of $\int_\rho \boldsymbol{F}\cdot d\boldsymbol{s}$?
A) 13 B) -13 C) 13/2 D) -13/2 E) NOTA

14. Suppose $f(x,y) = 24\sin x \cot y + e^{2x} - y$. Let A = f_{xy} and B = f_{yx} .
Now calculate the value of A – B.
A) $24\cos x \sin y$ B) $e^{2x} - 1$ C) 12 D) $24\sin x - 24\cot y$ E) NOTA

15. Consider a solid torus obtained as follows: Take the unit disc centered at (3, 0, 0) and lying entirely in the xz-plane; then rotate that disc around the z-axis. Compute the volume of the resulting torus.
A) $4\pi^2$ B) $6\pi^2$ C) $8\pi^2$ D) $10\pi^2$ E) NOTA

16. Whose theorem says that the order of integration can be switched without changing the final value of a double integral?

Namely: $$\int_A \left(\int_B f(x,y)\,\mathrm{d}y\right)\mathrm{d}x = \int_B \left(\int_A f(x,y)\,\mathrm{d}x\right)\mathrm{d}y = \int_{A\times B} f(x,y)\,\mathrm{d}(x,y),$$

A) Cauchy B) Fubini C) Gauss D) Leibniz E) NOTA

17. For a given vector field $\mathbf{F} = e^{xy}\mathbf{i} + e^{x+y}\mathbf{j}$, calculate the curl at the point (x,y) = (0,1). Is this a gradient vector field?
A) 1- e; yes B) 0; yes C) 1+ e; no D) e; no E) NOTA

For **questions 18-20**, let S be the level surface of $F(x,y,z) = x\sin y + y\sin z = 0$.

18. Find the tangent plane to the surface S at (1, π, 0).
A) $x + \pi y = 0$ B) $x + y + z = 1 + \pi$ C) $y - \pi z = \pi$ D) $y + \pi z = 0$ E) NOTA

19. Which theorem would be used to show that, near the point (1, π, 0), the level surface S is the graph of a function $f(x,y)$?
A) Gradient Theorem B) Gȯdel's Theorem C) Implicit Function Theorem
D) Squeeze Theorem E) NOTA

20. Find the partial derivative $\frac{\partial f}{\partial x}(1,\pi)$.
A) -1 B) 0 C) 1 D) ∞ E) NOTA

21. Compute: $\lim_{n\to\infty}(\frac{1}{n}A^n)$,where A is the 2x2 matrix $\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}$. Your answer should be another 2x2 matrix. Find the sum of the four entries.
A) 0 B) 1 C) 10 D) 100 E) NOTA

22. In the spherical coordinate system, recall that: $x = \rho\sin\varphi\cos\theta$, $y = \rho\sin\varphi\sin\theta$, $z = \rho\cos\varphi$. What is the simplified form of $\left|\frac{\partial(x,y,z)}{\partial(\rho,\theta,\varphi)}\right|$?
A) $\rho^2\sin\varphi$ B) $\rho^2(\cos\varphi)^2\sin\varphi$ C) $\rho\sin\theta\cos\theta$ D) $\rho^2\sin\varphi\cos\theta$ E) NOTA

For questions 23 and 24, suppose that $\vec{a}, \vec{b}, \vec{c}$ are vectors in R^3. You are given that the following vector identity always holds:

$$\vec{a}\times(\vec{b}\times\vec{c}) = \vec{b}(\vec{a}\cdot\vec{c}) - \vec{c}(\vec{a}\cdot\vec{b})$$

You are also given that the three vectors satisfy the following conditions:
$\vec{a}\times(\vec{a}\times\vec{b}) = \vec{0}$, $\vec{b}\cdot\vec{b} = 1$, $\vec{a}\cdot\vec{b} = -2$, $\vec{b}\cdot\vec{c} = 2$

23. Calculate the value of $\vec{a}\times\vec{b}$.
A) $\overrightarrow{-4}$ B) $\overrightarrow{-2}$ C) $\vec{0}$ D) $\vec{4}$ E) NOTA

24. Calculate the value of $\vec{a}\cdot\vec{c}$.
A) -4 B) -2 C) 0 D) 4 E) NOTA

25. Find the extreme points of $f(x,y,z) = x + y + z$ subject to the conditions $x^2 + y^2 = 2$ and $x + z = 1$.
A) (1, 0, 0); (-1, $\sqrt{2}$, 1) B) ($\sqrt{2}$, 0, 1-$\sqrt{2}$); (1, $\sqrt{2}$, 0) C) (1, 1, 0); (-1, 1, 2)
D) (0, -$\sqrt{2}$, 1); (0, $\sqrt{2}$, 1) E) NOTA

26. Let $\mathbf{F}(x,y,z) = (2xyz + \sin x)\mathbf{i} + x^2z\mathbf{j} + x^2y\mathbf{k}$. If possible, find a function *f* such that **F**= ∇f.
A) $f = x^2yz + \cos x + C$ B) $x^2yz - \cos x + \sin y + C$ C) $xyz + C$
D) Impossible E) NOTA

27. Calculate the value of:

$$\int_0^1\int_0^x\int_0^y (y + xz)\,dz\,dy\,dx$$

A) 11/30 B) 17/60 C) 1/3 D) 23/60 E) NOTA

28. Find the area of the two-dimensional region $x^2 - 2xy + 3y^2 \le 1$.
Hint: Note that we can rewrite the inequality as $(x - y)^2 + 2y^2 \le 1$.
A) $\frac{\pi}{4}$ B) $\frac{\pi\sqrt{2}}{2}$ C) $\frac{\pi}{2}$ D) $\frac{\pi\sqrt{3}}{4}$ E) NOTA

29. Use Stokes' theorem to evaluate the line integral:

$$\int_C -y^3dx + x^3dy - z^3dz$$

Where C is the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 1$, and the orientation on C corresponds to counterclockwise motion in the xy-plane.
A) 0 B) $\pi/2$ C) π D) $3\pi/2$ E) NOTA

30. Use Gauss' divergence theorem to evaluate $\int_{\partial W}(x^2 + y + z)dS$, where W is the solid ball $x^2 + y^2 + z^2 \le 1$.
A) 0 B) $2\pi/3$ C) $4\pi/3$ D) $8\pi/3$ E) NOTA