#0 Mu Bowl
MAΘ National Convention 2016

$$A = \int_{-2016}^{2016} x^2 \sin x^5 \, dx \qquad B = \int_{-\infty}^{\infty} e^{-x^2} \, dx \qquad C = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

Compute ABC.

#0 Mu Bowl
MAΘ National Convention 2016

$$A = \int_{-2016}^{2016} x^2 \sin x^5 \, dx \qquad B = \int_{-\infty}^{\infty} e^{-x^2} \, dx \qquad C = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

Compute ABC.

#1 Mu Bowl
MAΘ National Convention 2016

Let R be the region bound by the graphs of $y = \sqrt{x}$ and $x = 2y$.

A = the volume when R is revolved about the x-axis.

B = the volume when R is revolved about the y-axis.

C = the volume when R is revolved about $y = -1$.

D = the volume when R is revolved about $x = 4$.

Compute $A + B + C + D$.

#1 Mu Bowl
MAΘ National Convention 2016

Let R be the region bound by the graphs of $y = \sqrt{x}$ and $x = 2y$.

A = the volume when R is revolved about the x-axis.

B = the volume when R is revolved about the y-axis.

C = the volume when R is revolved about $y = -1$.

D = the volume when R is revolved about $x = 4$.

Compute $A + B + C + D$.

#2 Mu Bowl
MAΘ National Convention 2016

$$A = \lim_{x\to 1} \frac{3x^5 - 9x^3 + 13x^2 - 14x + 7}{5x^4 - 3x^3 - 13x^2 + 15x - 4}$$

$$B = \lim_{x\to 2} \frac{\sqrt{x+2} - 2}{\sqrt{3x+3} - 3}$$

$$C = \lim_{h\to 0} \frac{\sin\left(\frac{\pi}{3} + h\right) - \sin\left(\frac{\pi}{3}\right)}{h}$$

$$D = \lim_{x\to\infty} \left(\sqrt{x^2 + 5x} - \sqrt{x^2 - 3x}\right)$$

$$E = \lim_{x\to\infty} \left(1 - \frac{3}{x}\right)^{2x}$$

Compute $ABCD \ln E$.

#2 Mu Bowl
MAΘ National Convention 2016

$$A = \lim_{x\to 1} \frac{3x^5 - 9x^3 + 13x^2 - 14x + 7}{5x^4 - 3x^3 - 13x^2 + 15x - 4}$$

$$B = \lim_{x\to 2} \frac{\sqrt{x+2} - 2}{\sqrt{3x+3} - 3}$$

$$C = \lim_{h\to 0} \frac{\sin\left(\frac{\pi}{3} + h\right) - \sin\left(\frac{\pi}{3}\right)}{h}$$

$$D = \lim_{x\to\infty} \left(\sqrt{x^2 + 5x} - \sqrt{x^2 - 3x}\right)$$

$$E = \lim_{x\to\infty} \left(1 - \frac{3}{x}\right)^{2x}$$

Compute $ABCD \ln E$.

#3 Mu Bowl
MAΘ National Convention 2016

Let $f(x) = x^2$. $S(a,b)$ is defined to be the sum of the slope(s) of the line(s) tangent to f that pass through the point (a,b). If no tangent line to f pass through (a,b), then $S(a,b) = 0$. For example:

$S(1,1) = 2$, since $y = 2x - 1$ is the only line tangent to f that passes through $(1,1)$;

$S(0,1) = 0$, since no tangent line to f pass through $(0,1)$;

$S(0,-1) = 0$, since both $y = 2x - 1$ and $y = -2x - 1$ are tangent to f and pass through $(0,-1)$.

Compute

$$\sum_{b=0}^{100} \sum_{a=0}^{10} S(a,b)$$

#3 Mu Bowl
MAΘ National Convention 2016

Let $f(x) = x^2$. $S(a,b)$ is defined to be the sum of the slope(s) of the line(s) tangent to f that pass through the point (a,b). If no tangent line to f pass through (a,b), then $S(a,b) = 0$. For example:

$S(1,1) = 2$, since $y = 2x - 1$ is the only line tangent to f that passes through $(1,1)$;

$S(0,1) = 0$, since no tangent line to f pass through $(0,1)$;

$S(0,-1) = 0$, since both $y = 2x - 1$ and $y = -2x - 1$ are tangent to f and pass through $(0,-1)$.

Compute

$$\sum_{b=0}^{100} \sum_{a=0}^{10} S(a,b)$$

#4 Mu Bowl
MAΘ National Convention 2016

A = the area enclosed by the rectangular equation $4x^2 + 9y^2 - 16x - 54y + 61 = 0$.

B = the area enclosed by the polar equation $r = 3\sin 3\theta$.

C = the area enclosed by the parametric equation $x = \sin t\,, y = 3\sin 2t$.

Compute $\frac{A}{B} + C$.

#4 Mu Bowl
MAΘ National Convention 2016

A = the area enclosed by the rectangular equation $4x^2 + 9y^2 - 16x - 54y + 61 = 0$.

B = the area enclosed by the polar equation $r = 3\sin 3\theta$.

C = the area enclosed by the parametric equation $x = \sin t\,, y = 3\sin 2t$.

Compute $\frac{A}{B} + C$.

#5 Mu Bowl
MAΘ National Convention 2016

A. Let $f_1(x) = \sin x + \cos x$, approximate $f_1(0.2)$ using the line tangent to $f_1(x)$ at $x = 0$.

B. Let $f_2(x) = \sqrt{25 - x^2}$, approximate $f_2(3.1)$ using the line tangent to $f_2(x)$ at $x = 3$.

C. Let $f_3(x) = \frac{4}{x^2+3}$, approximate $f_3(-1.1)$ using the line tangent to $f_3(x)$ at $x = -1$.

D. Let $f_4(x) = x^3 - 6x^2 + 12x$, approximate $f_4(0.9)$ using the line tangent to $f_4(x)$ at $x = 1$.

Define $E(X)$ as a function of part X of this problem. The value of $E(X)$ is the value of the approximation if part X results in an overestimate, or the negation of the approximation if part X results in an underestimate. Compute $E(A) + E(B) + E(C) + E(D)$.

#5 Mu Bowl
MAΘ National Convention 2016

A. Let $f_1(x) = \sin x + \cos x$, approximate $f_1(0.2)$ using the line tangent to $f_1(x)$ at $x = 0$.

B. Let $f_2(x) = \sqrt{25 - x^2}$, approximate $f_2(3.1)$ using the line tangent to $f_2(x)$ at $x = 3$.

C. Let $f_3(x) = \frac{4}{x^2+3}$, approximate $f_3(-1.1)$ using the line tangent to $f_3(x)$ at $x = -1$.

D. Let $f_4(x) = x^3 - 6x^2 + 12x$, approximate $f_4(0.9)$ using the line tangent to $f_4(x)$ at $x = 1$.

Define $E(X)$ as a function of part X of this problem. The value of $E(X)$ is the value of the approximation if part X results in an overestimate, or the negation of the approximation if part X results in an underestimate. Compute $E(A) + E(B) + E(C) + E(D)$.

#6 Mu Bowl
MAΘ National Convention 2016

$$A = \sum_{n=0}^{\infty} \frac{2n}{n!}$$
$$B = \sum_{n=0}^{\infty} \frac{2^n}{n!}$$
$$C = \sum_{n=0}^{\infty} \frac{n^2}{n!}$$

Compute $A + B + C$.

#6 Mu Bowl
MAΘ National Convention 2016

$$A = \sum_{n=0}^{\infty} \frac{2n}{n!}$$
$$B = \sum_{n=0}^{\infty} \frac{2^n}{n!}$$
$$C = \sum_{n=0}^{\infty} \frac{n^2}{n!}$$

Compute $A + B + C$.

#7 Mu Bowl
MAΘ National Convention 2016

A = $\int_0^6 (36x - x^3)\, dx$

B = approximation of A using midpoint Riemann sum with 3 equal intervals.

C = approximation of A using trapezoidal rule with 3 equal intervals.

D = approximation of A using Simpson's Rule with $n = 6$.

Compute $A + B - C - D$.

#7 Mu Bowl
MAΘ National Convention 2016

A = $\int_0^6 (36x - x^3)\, dx$

B = approximation of A using midpoint Riemann sum with 3 equal intervals.

C = approximation of A using trapezoidal rule with 3 equal intervals.

D = approximation of A using Simpson's Rule with $n = 6$.

Compute $A + B - C - D$.

#8 Mu Bowl
MAΘ National Convention 2016

A 200mL container is lined with a membrane that allows water to seep through at a rate proportional to the concentration of water in the container. The container is initially full of a 20% solution of alcohol in water. As water seeps out of the container, it is instantly replaced with a 20% solution of alcohol in water and remixed evenly. If 400mL of water seeps through in 2 hours, how many hours does it take in total for the concentration of alcohol inside the container to reach at least 90%?

#8 Mu Bowl
MAΘ National Convention 2016

A 200mL container is lined with a membrane that allows water to seep through at a rate proportional to the concentration of water in the container. The container is initially full of a 20% solution of alcohol in water. As water seeps out of the container, it is instantly replaced with a 20% solution of alcohol in water and remixed evenly. If 400mL of water seeps through in 2 hours, how many hours does it take in total for the concentration of alcohol inside the container to reach at least 90%?

#9 Mu Bowl
MAΘ National Convention 2016

Let $f(x) = \frac{1}{2}x^2 + bx + c$, where $b = 3, c = -8$.

A = the greater of two zeros of $f(x)$.

B = the rate of change of the greater of two zeros of $f(x)$ if b is changing at 2 per second and c is constant at the moment when $b = 3, c = -8$.

C = the rate of change of the greater of two zeros of $f(x)$ if b is constant and c is changing at 2 per second at the moment when $b = 3, c = -8$.

D = the rate of change of the greater of two zeros of $f(x)$ if b and c are changing at 2 per second at the moment when $b = 3, c = -8$.

Compute $A + B + C + D$.

#9 Mu Bowl
MAΘ National Convention 2016

Let $f(x) = \frac{1}{2}x^2 + bx + c$, where $b = 3, c = -8$.

A = the greater of two zeros of $f(x)$.

B = the rate of change of the greater of two zeros of $f(x)$ if b is changing at 2 per second and c is constant at the moment when $b = 3, c = -8$.

C = the rate of change of the greater of two zeros of $f(x)$ if b is constant and c is changing at 2 per second at the moment when $b = 3, c = -8$.

D = the rate of change of the greater of two zeros of $f(x)$ if b and c are changing at 2 per second at the moment when $b = 3, c = -8$.

Compute $A + B + C + D$.

#10 Mu Bowl
MAΘ National Convention 2016

f is a twice differentiable function over all real numbers. The table below shows the value of f and f' at select values on the interval $[0, 10]$.

x	0	2	4	6	8	10
$f(x)$	-3	2	4	-4	0	3
$f'(x)$	1	-1	-2	2	-1	4

A = the minimum number of zeros of f on $(0, 10)$.

B = the minimum number of local maxima of f on $(0, 10)$.

C = the minimum number of local minima of f on $(0, 10)$.

D = the minimum number of points of inflection of f on $(0, 10)$.

Compute $A^2 + B^2 + C^2 + D^2$.

#10 Mu Bowl
MAΘ National Convention 2016

f is a twice differentiable function over all real numbers. The table below shows the value of f and f' at select values on the interval $[0, 10]$.

x	0	2	4	6	8	10
$f(x)$	-3	2	4	-4	0	3
$f'(x)$	1	-1	-2	2	-1	4

A = the minimum number of zeros of f on $(0, 10)$.

B = the minimum number of local maxima of f on $(0, 10)$.

C = the minimum number of local minima of f on $(0, 10)$.

D = the minimum number of points of inflection of f on $(0, 10)$.

Compute $A^2 + B^2 + C^2 + D^2$.

#11 Mu Bowl
MAΘ National Convention 2016

f and g are invertible functions that are locally differentiable at 1, 2, 3, and 4. The table below shows the evaluation of those functions and their derivatives. Let

$h_1(x) = f(2x) + g(3x)$

$h_2(x) = f\left(f\big(f(x)\big)\right)$

$h_3(x) = xf(x^2)$

$h_4(x) = f(x)g^{-1}(x)$

$h_5(x) = f(x)\big(g(x)\big)^{-1}$

$h_6(x) = f\big(f^{-1}(x)\big)$

x	f	f'	g	g'
1	1	5	4	-3
2	4	2	3	1
3	2	-1	1	3
4	3	3	2	-6

Compute $h_1'(1) + h_2'(3) + h_3'(2) + h_4'(4) + h_5'(3) + h_6'(4)$.

#11 Mu Bowl
MAΘ National Convention 2016

f and g are invertible functions that are locally differentiable at 1, 2, 3, and 4. The table below shows the evaluation of those functions and their derivatives. Let

$h_1(x) = f(2x) + g(3x)$

$h_2(x) = f\left(f\big(f(x)\big)\right)$

$h_3(x) = xf(x^2)$

$h_4(x) = f(x)g^{-1}(x)$

$h_5(x) = f(x)\big(g(x)\big)^{-1}$

$h_6(x) = f\big(f^{-1}(x)\big)$

x	f	f'	g	g'
1	1	5	4	-3
2	4	2	3	1
3	2	-1	1	3
4	3	3	2	-6

Compute $h_1'(1) + h_2'(3) + h_3'(2) + h_4'(4) + h_5'(3) + h_6'(4)$.

#12 Mu Bowl
MAΘ National Convention 2016

A = $\int_{-2}^{1} x\sqrt{x+3}\, dx$

B = $\int_{0}^{3} x\sqrt{9-x^2}\, dx$

C = $\int_{0}^{2} xe^{x/2}\, dx$

D = $\int_{0}^{\frac{\pi}{2}} \sin 2x\, e^{\sin x}\, dx$

Compute $5A - B + C - 2D$.

#12 Mu Bowl
MAΘ National Convention 2016

A = $\int_{-2}^{1} x\sqrt{x+3}\, dx$

B = $\int_{0}^{3} x\sqrt{9-x^2}\, dx$

C = $\int_{0}^{2} xe^{x/2}\, dx$

D = $\int_{0}^{\frac{\pi}{2}} \sin 2x\, e^{\sin x}\, dx$

Compute $5A - B + C - 2D$.

#13 Mu Bowl
MAΘ National Convention 2016

Let $f(x) = x^2e^x$. Let $f^{(k)}(x)$ denote the k^{th} derivative of $f(x)$, then the sum

$$\sum_{k=1}^{20} f^{(k)}(x)$$

can be expressed as $Ax^2e^x + Bxe^x + Ce^x$, where A, B, C are real. Compute $A + B + C$.

#13 Mu Bowl
MAΘ National Convention 2016

Let $f(x) = x^2e^x$. Let $f^{(k)}(x)$ denote the k^{th} derivative of $f(x)$, then the sum

$$\sum_{k=1}^{20} f^{(k)}(x)$$

can be expressed as $Ax^2e^x + Bxe^x + Ce^x$, where A, B, C are real. Compute $A + B + C$.

#14 Mu Bowl
MAΘ National Convention 2016

Let ℓ_1 be the line described by $x = \frac{y-3}{2} = \frac{z-3}{2}$, and ℓ_2 be the line $\frac{x}{2} = \frac{y+1}{3} = \frac{z+4}{6}$.

Particle A moves along ℓ_1 at a constant speed of 6 units per second. Particle B moves along ℓ_2 at a constant speed of 7 units per second. At $t = 0$, particle A is at point $(0, 3, 3)$, and particle B is at the point $(0, -1, -4)$. Both particles are moving in the direction such that their x-coordinates are increasing. At time $t = T$ seconds, the two particles make their closest approach to each other, where they are D units apart. Compute $T^2 + D^2$.

#14 Mu Bowl
MAΘ National Convention 2016

Let ℓ_1 be the line described by $x = \frac{y-3}{2} = \frac{z-3}{2}$, and ℓ_2 be the line $\frac{x}{2} = \frac{y+1}{3} = \frac{z+4}{6}$.

Particle A moves along ℓ_1 at a constant speed of 6 units per second. Particle B moves along ℓ_2 at a constant speed of 7 units per second. At $t = 0$, particle A is at point $(0, 3, 3)$, and particle B is at the point $(0, -1, -4)$. Both particles are moving in the direction such that their x-coordinates are increasing. At time $t = T$ seconds, the two particles make their closest approach to each other, where they are D units apart. Compute $T^2 + D^2$.