Note: For all questions, answer "(E) NOTA " means none of the above answers is correct.

1. Let $a_n = 2a_{n-1} - a_{n-2}$, $a_0 = 3$, and $a_1 = 7$. Evaluate: a_{23}.

   (A) 75 (B) 87 (C) 91 (D) 95 (E) NOTA

2. The first six terms of the polynomial sequence $a_n = p(n)$ are 5, 4, 1, -1, 1, 10. What is the least possible degree of polynomial p?

   (A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

3. Evaluate: $\frac{26}{11+\frac{26}{11+\cdots}}$

   (A) 2 (B) $2+\sqrt{3}$ (C) 3 (D) $3+\sqrt{2}$ (E) NOTA

4. $f(x) = e^{\left(\frac{x}{4}\right)}$. Evaluate: $\sum_{n=0}^{\infty} f^{(n)}(n)$

   (A) $\frac{4}{\sqrt[4]{e}}$ (B) $\frac{4}{4-\sqrt[4]{e}}$ (C) $\frac{1}{4-\sqrt[4]{e}}$ (D) $4\sqrt[4]{e}$ (E) NOTA

5. For what values of p, does $\sum_{n=2}^{\infty} \frac{1}{(n \ln^p n)}$ converge?

   (A) $p > 0$ (B) $p > 1$ (C) $p > 2$ (D) $p > e$ (E) NOTA

6. Which test determines the convergence or divergence of $\sum_{n=2}^{\infty} \frac{1}{1-n}$?

   (A) Alternating series test (B) Ratio Test
   (C) Limit comparison to $-\frac{1}{n}$ (D) Direct comparison to $-\frac{1}{n}$ (E) NOTA

7. Which test **cannot** determine the convergence or divergence of $\sum_{n=1}^{\infty} \left(-\frac{1}{n}\right)^n$?

   (A) Alternating series test (B) Ratio Test
   (C) Limit comparison to $-\frac{1}{n}$ (D) Root Test (E) NOTA

8. What is the radius of convergence of $\sum_{n=1}^{\infty} \frac{(xn)^n}{n!}$?

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) e (E) NOTA

9. What is the interval(s) of convergence of $\sum_{n=1}^{\infty} \frac{5}{n}\left(\frac{2x-1}{x+2}\right)^n$?

(A) $\left[\frac{1}{7}, 1\right)$ (B) $\left(\frac{1}{5}, 2\right)$ (C) $\left[\frac{1}{3}, 3\right)$ (D) $[1, 2)$ (E) NOTA

10. Let C and D refer to a convergent and a divergent series of reals respectively and let $A \cdot B$, refer to the series $\sum_{i=0}^{\infty} a_n b_n$, where $A = a_0, a_1, \ldots, a_n$, and $B = b_0, b_1, \ldots, b_n$. Which of following statements is always necessarily true.

I. $C \cdot C$ converges
II. $C \cdot D$ converges
III. $C \cdot D$ diverges
IV. $D \cdot D$ diverges

(A) I only (B) I, II, IV only (C) III, IV only (D) IV only (E) NOTA

11. What fraction of the first 100! elements of the Fibonacci sequence are divisible by 13? The first two terms of the sequence are both 1.

(A) $\frac{1}{6}$ (B) $\frac{1}{7}$ (C) $\frac{1}{13}$ (D) $\frac{1}{14}$ (E) NOTA

12. Describe the convergence or divergence of $\sum_{n=1}^{\infty} a_n$, where $a_n = \frac{e^{2\pi i n/3}}{n}$

(A) Converges absolutely
(B) Converges conditionally
(C) The real component converges, but the imaginary component diverges
(D) The imaginary component converges, but the real component diverges
(E) NOTA

13. A unit cube has cubes of side length $\frac{1}{3}$ placed on the middle of each of its faces. In the middle of each uncovered face of the newly added cubes a cube of side length $\frac{1}{9}$ is placed. In the middle of each open face of these a cube of side length $\frac{1}{27}$ is placed and so on. What is the volume of the resulting fractal?

(A) $\frac{14}{11}$ (B) $\frac{26}{21}$ (C) $\frac{27}{22}$ (D) $\frac{751}{594}$ (E) NOTA

14. Evaluate: $\lim_{n\to\infty} \sum_{i=1}^{n^2} \frac{e^{i/n}}{ne^n}$

(A) $\frac{1}{e}$ (B) 1 (C) e (D) e^2 (E) NOTA

15. What is the sum of all three digit multiples of 12?

(A) 40,404 (B) 40,836 (C) 41,400 (D) 41,832 (E) NOTA

16. How many complex values are there for g_6, given that g is a geometric sequence, $g_4 = 4$ and $g_{10} = 256$?

(A) 2 (B) 4 (C) 3 (D) 6 (E) NOTA

17. Sam uploads a funny cat video and shares it with 4 of her friends. The next day everyone who has seen the video shares it with 4 new people and the same thing happens each day after that. After how many days have one million people seen the video?

(A) 8 (B) 9 (C) 10 (D) 11 (E) NOTA

18. For some c, where $0 < c < 1$, let $f(x) = c^x \cos(x)$, $A = \int_0^{2\pi} f(x)dx$, and $B = \int_\pi^{3\pi} f(x)dx$. Express $\int_0^\infty f(x)dx$, in terms of A and B.

(A) $\frac{A^2}{A-B}$ (B) $\frac{A^3}{(A+B)^2}$ (C) $\frac{A^3}{A^2-B^2}$ (D) $\frac{A^3}{(A-B)^2}$ (E) NOTA

19. What is the Maclaurin series for $\cos^2 x$?

(A) $\sum_{n=0}^{\infty} \frac{(-x)^{4n}}{((2n)!)^2}$ (B) $\sum_{n=0}^{\infty} \frac{x^{4n}}{(4n)!}$ (C) $\sum_{n=0}^{\infty} \frac{(-2x)^{2n}}{(2n)!}$ (D) $\frac{1}{2} + \sum_{n=0}^{\infty} \frac{(-1)^n (2x)^{2n}}{2(2n)!}$

(E) NOTA

20. What is the sum of the least 14 positive perfect cubes?

(A) 8,100 (B) 11,025 (C) 12,225 (D) 14,700 (E) NOTA

21. Evaluate: $\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$

(A) $\frac{8}{13}$ (B) $\frac{11}{18}$ (C) $\frac{13}{36}$ (D) $\frac{47}{180}$ (E) NOTA

22. Let $a_n = {a_{n-1}}^{1/a_{n-1}}$ and $f(x) = \lim_{n\to\infty} a_n$, where $a_0 = x > 0$. Which of the following correctly defines f as a real valued function over the positive reals?

(A) $f(x) = 1, \quad 0 < x \leq 1$ (B) $f(x) = \lfloor x \rfloor, \quad x > 0$

(C) $f(x) = \begin{cases} 1, & 0 < x \leq 1 \\ 0, & 1 < x \end{cases}$ (D) $f(x) = \begin{cases} 0, & 0 < x < 1 \\ 1, & 1 \leq x \end{cases}$ (E) NOTA

23. Let $f(x)$ be a monotonically decreasing positive function with y-intercept 1. Which of the following guarantees the volume of the region bounded by rotating $f(x)$ about the x-axis is finite?

I. $\sum_{n=0}^{\infty} f(n) < \infty$

II. $\sum_{n=0}^{\infty} f^2(n) < \infty$

III. $\sum_{n=0}^{\infty} f^3(n) < \infty$

(A) I, II (B) II (C) II, III (D) I, II, III (E) NOTA

24. How many increasing sequences of positive integers less than or equal to 100 are there?

(A) $\binom{100}{2} - 1$ (B) $2^{100} - 1$ (C) $\frac{100!}{2} - 1$ (D) $100! - 1$ (E) NOTA

25. In what quadrant of the Argand plane does the complex value $\prod_{n=1}^{\infty} e^{i/n^2}$ lie?

(A) I (B) II (C) III (D) IV (E) NOTA

26. Estimate $\tan(\pi/3)$, using the 2nd degree Taylor polynomial for tan(x) centered at $\pi/4$.

(A) $\frac{\pi}{3}$ (B) $1 + \frac{\pi}{24} + \frac{\pi^2}{96}$ (C) $1 + \frac{\pi}{12} + \frac{\pi^2}{144}$ (D) $1 + \frac{\pi}{6} + \frac{\pi^2}{72}$ (E) NOTA

27. What is the Taylor series for $\ln x$ centered at e?

(A) $\sum_{n=1}^{\infty} \frac{(x-e)^n}{n!}$ (B) $\sum_{n=1}^{\infty} \frac{(x-e)^n}{n}$ (C) $\sum_{n=1}^{\infty} \frac{(-\frac{x}{e}+1)^n}{n}$ (D) $-\sum_{n=1}^{\infty} \frac{(-\frac{x}{e}+1)^n}{n}$ (E) NOTA

28. Solve for x. $x = \sqrt{2550 + \sqrt{2550 + \sqrt{2550 + \sqrt{2550 + \cdots}}}}$

(A) 48 (B) 49 (C) 50 (D) 51 (E) NOTA

29. Evaluate: $\int_1^{\infty} \left(e^{-\lfloor x \rfloor} - e^{-x}\right) dx$

(A) $\frac{1}{e}$ (B) $\frac{1}{e^2-e}$ (C) $\frac{e-1}{e}$ (D) $\frac{e}{e-1}$ (E) NOTA

30. What fraction is equivalent to the following repeating decimal 0.814814 …?

(A) $\frac{22}{27}$ (B) $\frac{30}{37}$ (C) $\frac{90}{111}$ (D) $\frac{268}{333}$ (E) NOTA