1. D
2. B
3. A
4. B
5. B
6. C
7. C
8. B
9. E
10. E
11. B
12. B
13. A
14. B
15. C
16. C
17. B
18. C
19. D
20. B
21. B
22. D
23. A
24. B
25. B
26. D
27. D
28. D
29. B
30. A

Solutions

1. The recurrence relation makes a_n an arithmetic sequence. Thus $a_n = (a_1 - a_0)n + 3 = 4n + 3$ $f_0 = 3$. $a_{23} = 4 \times 23 + 3 = 95$.

2. The degree of the polynomial is number of finite differences that must be taken before the (infinite) sequence is constant (see http://en.wikipedia.org/wiki/Newton_series).
5, 4, 1, -1, 1, 10
-1, -3, -2, 2, 9
-2, 1, 4, 7
3, 3, 3
Degree: 3

3.
$$x = \frac{26}{11+x}$$
$$x^2 + 11x - 26 = 0$$
$$x = 2, -13$$
2 is the only value greater than 0.

4. $f^{(n)}(n) = e^{\frac{n}{4}}/4^n = \left(\sqrt[4]{e}/4\right)^n$. $\sum_{n=0}^{\infty} f^{(n)}(n) = \frac{1}{1-\sqrt[4]{e}/4} = \frac{4}{4-\sqrt[4]{e}}$

5. $\int_2^{\infty} \frac{1}{(x \ln^p x)} dx = \int_{\ln 2}^{\infty} u^{-p} du$, which converges for $p > 1$.

6. Limit comparison test: $0 \geq \frac{1}{1-n} > -\frac{1}{n}$ and $\lim_{n\to\infty} -\frac{n}{1-n} = 1 > 0$

7. Limit comparison test: $\lim_{n\to\infty} \left(-\frac{1}{n}\right)^n / \left(-\frac{1}{n}\right) = \left(-\frac{1}{n}\right)^{n-1} = 0$

8. One could use ratio test, but Stirling's approximation is simpler. $n! \approx \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$. The series is thus approximately $\sum_{n=1}^{\infty} \frac{(xn)^n}{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n} = \sum_{n=1}^{\infty} \frac{(ex)^n}{\sqrt{2\pi n}}$. Radius $= \frac{1}{e}$.

9. Converges when: $-1 \leq \frac{2x-1}{x+2} < 1$
Solve for edges: $-1 = \frac{2x-1}{x+2}$ OR $\frac{2x-1}{x+2} = 1$
$x = -1/3$ or $x = 3$. $-1/3 \leq x < 3$.

10. None are necessarily true. Here are counter examples
    I. $C = \frac{(-1)^n}{\sqrt{n}}, C \cdot C = \frac{1}{n}$ diverges
    II. $C = 2^{-n},\ D = 2^n, C \cdot D = 1$ diverges
    II. $C = 2^{-n},\ D = 1, C \cdot D = 2^{-n}$ converges
    II. $D = \frac{1}{n}, D \cdot D = \frac{1}{n^2}$ converges

11. Sequence mod 13 is 1, 1, 2, 3, 5, 8, 0 , 8, 8, 3, 11, 1, 12, 0,...
    (12 is congruent to -1 so the next 14 are 13 minus the first 14). Repeats every 7.

12. $|a_n| = 1/n$, so it does not converge absolutely.
    The real component of the sum is $-\frac{1}{2} + \left(-\frac{1}{4}\right) + \frac{1}{3} + \left(-\frac{1}{8}\right) + \left(-\frac{1}{10}\right) + \frac{1}{6} + \cdots$
    $=\left(-\frac{3}{4}\right) + \frac{1}{3} + \left(-\frac{9}{40}\right) + \frac{1}{6} + \cdots$, which is a diminishing alternating series and converges.
    and the same for the imaginary component.

13. $1+6 \times \left(\frac{1}{3}\right)^3 + 6 \times 5 \times \left(\frac{1}{3}\right)^6 + 6 \times 5^2 \times \left(\frac{1}{3}\right)^9 + \cdots$

$$= 1 + \frac{2}{9} + \frac{2}{9} \times \left(\frac{5}{27}\right) + \frac{2}{9} \times \left(\frac{5}{27}\right)^2 + \cdots$$
$$= 1 + \frac{2/9}{(1 - 5/27)}$$
$$= \frac{14}{11}$$

14. $\lim_{n\to\infty} \sum_{i=1}^{n^2} \frac{e^{i/n}}{ne^n} = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n^2} e^{i/n-n} = \int_{-\infty}^{0} e^x dx = 1$

15. $\left(\frac{108+996}{2}\right)\left(\frac{996-108}{12} + 1\right) = 552 \times 75 = 41{,}400$

16. 6 complex roots of any 6th power, but only three distinct roots for the square of the common ratio.

17. On the n^{th} day, 5^n people have seen the video $\log_5 1{,}000{,}000 = 6 \log_5 10 \approx 8.6$. It will pass 1,000,000 on the 9th day.

18. $\frac{f(x+\pi)}{f(x)} = \frac{c^{x+\pi}\cos(x+\pi)}{c^x\cos(x)} = -c^\pi > -1$, Then we wish to find the sum of an infinite geometric series with $r = -c^\pi$, and $a_0 = \int_0^\pi f(x)dx$.
We have $r = \frac{f(x+\pi)}{f(x)} = \frac{B}{A}$, and $A = a_0(1+r)$, so $a_0 = \frac{A}{1+r} = \frac{A}{1+\frac{B}{A}}$.
The sum of the series is $\frac{a_0}{1-r} = \frac{A}{\left(1+\frac{B}{A}\right)\left(1-\frac{B}{A}\right)} = \frac{A^3}{A^2-B^2}$.

19. $\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos(2x) = \frac{1}{2} + \sum_{n=0}^{\infty} \frac{(2x)^{2n}}{2(2n)!}$

20. $\left(\frac{14(14+1)}{2}\right)^2 = 35^2 = 11{,}025$

21. $\sum_{n=1}^{\infty} \frac{1}{n(n+3)} = \sum_{n=1}^{\infty} \frac{1}{3}\left(\frac{1}{n} - \frac{1}{n+3}\right) = \frac{1}{3}\sum_{n=1}^{3} \frac{1}{n} = \frac{11}{18}$

22. D is the answer by inspection

23. You want $\pi \int_0^\infty f(x)^2 < \infty$, which given that f is monotonically decreasing and has a finite y-intercept, is true if $\sum_{n=0}^{\infty} f^2(n) < \infty$, and since it's positive that is true if $\sum_{n=0}^{\infty} f(n) < \infty$ as well.

24. Since they are increasing, order is fixed and it's just the number of nonempty subsets of a hundred element set, which is $2^{100} - 1$.

25. $\prod_{n=1}^{\infty} e^{i/n^2} = e^{i\sum_{n=1}^{\infty} 1/n^2} = e^{i\pi^2/6} = e^{i(\pi/2+\epsilon)}$. Quadrant II.

26. $\tan'(x) = \sec^2 x$ and $\tan''(x) = 2\sec^2 x \tan x$

$$\tan\left(\frac{\pi}{3}\right) \approx \tan\left(\frac{\pi}{4}\right) + \tan'\left(\frac{\pi}{4}\right)\left(\frac{\pi}{3} - \frac{\pi}{4}\right) + \frac{1}{2}\tan''\left(\frac{\pi}{4}\right)\left(\frac{\pi}{3} - \frac{\pi}{4}\right)^2$$
$$= \tan\left(\frac{\pi}{4}\right) + \sec^2\left(\frac{\pi}{4}\right)\left(\frac{\pi}{12}\right) + \frac{1}{2}\left(2\sec^2\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{4}\right)\right)\left(\frac{\pi}{12}\right)^2$$
$$= 1 + \frac{\pi}{6} + \frac{\pi^2}{72}$$

27. $f^{(n)}(e) = \frac{(-1)^{n-1}(n-1)!}{e^n} \Rightarrow \frac{f^{(n)}(e)}{n!} = \frac{(-1)^{n-1}}{ne^n}$, so the series is $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{ne^n}(x-e)^n$

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$$=\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}\left(\frac{x}{e}-1\right)^{n}=-\sum_{n=1}^{\infty}\frac{1}{n}\left(-\frac{x}{e}+1\right)^{n}$$

28. $x=\sqrt{2550+x}\rightarrow 0=x^2-x-2550=(x-51)(x+50)\rightarrow x=51$ since $x>0$

29. $\int_1^{\infty}\left(e^{-\lfloor x\rfloor}-e^{-x}\right)dx=\sum_{i=1}^{\infty}e^{-i}-\int_1^{\infty}e^{-x}dx \quad =\frac{e^{-1}}{1-e^{-1}}-e^{-1}=\frac{1}{e^2-e}$

30. $0.\overline{814}=\frac{814}{999}=\frac{22}{27}$