For all questions, E "NOTA" means none of the above answers is correct.

1. Find the 10th nonzero term of the power series for $f(x)=\frac{x}{(x^2-1)^2}$ (expanding about $x = 0$).

   a) $10x^{10}$ b) $10x^{12}$ c) $10x^{15}$ d) $10x^{19}$ e) NOTA

2. Find the exact value (in radians) of $1-\frac{1}{3!}+\frac{1}{5!}-\ldots+\frac{(-1)^n}{(2n+1)!}+\ldots$

   a) Cos 1 b) Sin 1 c) Cos $\sqrt{2}$ d) Sin $\sqrt{2}$ e) NOTA

3. Compute $\sum_{n=0}^{\infty} n\left(\frac{1}{5}\right)^n$.

   a) $\frac{3}{25}$ b) $\frac{2}{5}$ c) $\frac{7}{25}$ d) $\frac{9}{25}$ e) NOTA

4. Evaluate $\frac{1}{1}+\frac{1}{2}-\frac{2}{3}+\frac{1}{4}+\frac{1}{5}-\frac{2}{6}+\frac{1}{7}+\frac{1}{8}-\frac{2}{9}+\ldots+\frac{1}{3n+1}+\frac{1}{3n+2}-\frac{2}{3n+3}+\ldots$

   a) ln 2 b) ln 3 c) e^2 d) e^3 e) NOTA

5. Let $f(x)=x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\frac{x^5}{5}$, and $g(x)=f^{-1}(x)$. Compute $g^{(3)}(0)$ (the 3rd derivative of g).

   a) 1 b) ln 2 c) e^3 d) ln 4 e) NOTA

6. For a given $x > 0$, let a_n be the sequence defined by $a_1 = x$ for $n = 1$ and $a_n = x^{a_{n-1}}$ for $n \geq 2$. Find the largest x for which $\lim_{n\to\infty} a_n$ converges.

   a) e b) $e^{1/e}$ c) e^e d) e^{-1} e) NOTA

7. Find the Maclaurin Series for 2^x.

   a) $\sum_{n=0}^{\infty}\frac{2^n}{n!}x^n$ b) $\sum_{n=0}^{\infty}\frac{2^n}{n!}$ c) $\sum_{n=0}^{\infty}\frac{\ln 2}{n!}x^n$ d) $\sum_{n=0}^{\infty}\frac{(\ln 2)^n}{n!}x^n$ e) NOTA

8. For the sequence, $S_n=\frac{1}{2},\frac{2}{3},\frac{3}{4},\frac{4}{5},\ldots,\frac{n}{n+1},\ldots$, what is $\lim_{n\to\infty} S_n$?

   a) 3 b) 2 c) 1 d) Does Not Exist e) NOTA

9. Find the interval of convergence for the series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n(3^n)}$.

a) $-1 < x < 5$ b) $-1 < x \le 5$ c) $-2 < x < 4$ d) $-2 \le x < 4$ e) NOTA

10. Determine the coefficient of $\left(x - \frac{\pi}{2}\right)^6$ in the Taylor series expansion about $\frac{\pi}{2}$ of $f(x) = \sin x$.

a) $\frac{1}{120}$ b) $\frac{-1}{120}$ c) $\frac{1}{720}$ d) $\frac{-1}{720}$ e) NOTA

11. The measures of the 7 angles of a convex heptagon are in arithmetic progression. If the measure of the smallest angle is 101°, then the largest angle, to the nearest degree, has what measure?

a) 108° b) 147° c) 156° d) 161° e) NOTA

12. $(-20)^2 + (-19)^2 + (-18)^2 + \ldots + n^2 + \ldots + 18^2 + 19^2 + 20^2 = ?$

a) 0 b) 420 c) 4900 d) 5740 e) NOTA

13. If $\sin\theta + \sin^2\theta + \sin^3\theta + \ldots = \frac{3}{2}$, then $\sin\theta = ?$

a) -3 b) $\frac{-3}{5}$ c) $\frac{3}{5}$ d) 3 e) NOTA

14. Evaluate: $\sum_{i=1}^{20} \int_{i+1}^{i+3} 3x^2 dx$

a) 7,999 b) 9,260 c) 22,780 d) 44,100 e) NOTA

15. Which of the answers below is equal to $\binom{20}{0}^2 + \binom{20}{1}^2 + \binom{20}{2}^2 + \ldots + \binom{20}{20}^2$?

a) $\binom{40}{20}$ b) $\binom{20}{10}^4$ c) $\frac{\binom{40}{20}}{\binom{20}{10}}$ d) $2^{20}\binom{20}{10}$ e) NOTA

16. Find the limit of the sequence $(\sqrt{2}-\sqrt{3}),(\sqrt{3}-\sqrt{4}),(\sqrt{4}-\sqrt{5}),...,\left(\sqrt{n+1}-\sqrt{n+2}\right),...$

a) 0 b) $\frac{-1}{4}$ c) $\frac{-1}{2}$ d) -1 e) NOTA

17. The sum of an infinite geometric series is 8. The sum of the cubes of the terms of the series is $\frac{512}{7}$. What is the 1st term of the series?

a) 6 b) 4.5 c) 4 d) 3.5 e) NOTA

18. Find the sum of $\frac{1}{7}+\frac{2}{7^2}+\frac{1}{7^3}+\frac{2}{7^4}+...$

a) $\frac{7}{48}$ b) $\frac{3}{16}$ c) $\frac{5}{32}$ d) $\frac{7}{72}$ e) NOTA

19. Evaluate $\sum_{n=0}^{\infty}\frac{2^{n-1}}{n!}$.

a) e^2 b) $2e$ c) $\frac{1}{2}e^2$ d) $\frac{1}{2}e$ e) NOTA

20. If $\sum_{n=1}^{\infty}|a_n|$ converges, then which of the following is (are) true?

I. $\sum_{n=1}^{\infty}a_n$ converges II. $\sum_{n=1}^{\infty}a_n$ is absolutely convergent III. $\sum_{n=1}^{\infty}-a_n$ converges

a) I only b) II only c) I & III only d) I, II, and III e) NOTA

21. Let $a_1=1$ and $a_{n+1}=\sqrt{4+3a_n}$ for all positive integers n. If we know the sequence $\{a_n\}$ converges, then $\lim_{n\to\infty}a_n=$

a) -1 b) 0 c) 2 d) 4 e) NOTA

22. Find the radius of convergence for $\sum_{n=0}^{\infty}\frac{n!}{10^n}x^n$.

a) ∞ b) $-1<x<1$ c) $-1\le x\le 1$ d) 0 e) NOTA

23. Compute $\sum_{i=0}^{\infty}i(1+i)p^{i-2}$ for $0<p<1$.

a) $\frac{2}{p}$ b) $\frac{2}{p(1-p)}$ c) $\frac{2}{p(1-p)^2}$ d) $\frac{2}{p(1-p)^3}$ e) NOTA

24. Evaluate $\lim_{n\to\infty} \sum_{k=n+1}^{2n} \frac{1}{k}$.

a) 0 b) ½ c) ln 2 d) $1 + e$ e) NOTA

25. At MAO nationals, there is a man called Paul, who has an infinite amount of time. This year he is walking continuously at a speed of $\frac{1}{1+t^2}$ ft/s, starting at time $t = 0$. If he continues to walk for an infinite amount of time, how far will he walk (in ft)?

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) π d) $\frac{3\pi}{2}$ e) NOTA

26. Let f be one of the solutions to the differential equation $f''(x) - 2xf'(x) - 2f(x) = 0$. Supposing that f has a Taylor expansion $f(x) = 1 + x + ax^2 + bx^3 + cx^4 + dx^5 + \ldots$ near the origin, find (a, b, c, d).

a) $\left(\frac{1}{4}, \frac{2}{5}, \frac{2}{3}, 1\right)$ b) $\left(\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}\right)$ c) $\left(1, \frac{2}{3}, \frac{1}{2}, \frac{4}{15}\right)$ d) $\left(\frac{1}{4}, \frac{2}{15}, \frac{1}{2}, \frac{3}{8}\right)$ e) NOTA

27. Consider a sequence given by $a_n = a_{n-1} + 3a_{n-2} + a_{n-3}$, where $a_0 = a_1 = a_2 = 1$. What is the remainder when a_{2013} is divided by 7?

a) 5 b) 4 c) 2 d) 1 e) NOTA

28. Evaluate: $\sum_{x=0}^{\infty} \sum_{y=0}^{\infty} \frac{1}{2^{x+y+|x-y|}}$

a) $\frac{4}{3}$ b) $\frac{4}{9}$ c) $\frac{20}{3}$ d) $\frac{20}{9}$ e) NOTA

29. We say that a number is *arithmetically sequenced* if the digits, in order from left to right, form an arithmetic sequence. Compute the number of 4-digit positive integers which are arithmetically sequenced.

a) 30 b) 40 c) 50 d) 70 e) NOTA

30. A sequence of letters rolls off the tongue if the following two conditions are met:
1) The sequence does not begin or end with two consecutive consonants.
2) No three consecutive letters are all consonants.

How many distinct permutations of “MATHEMATICS” roll off the tongue?

a) (10)9! b) (15)7! c) (10)6! d) (15)5! e) NOTA