1. D— Note that $\int f(x)dx = \frac{1}{2(1-x^2)} = \frac{1}{4}\left(\frac{1}{1+x}+\frac{1}{1-x}\right)$. These are geometric sums, so we have $\int f(x)dx = \frac{1}{4}\left(\sum_{k=0}^{\infty} x^k + \sum_{k=0}^{\infty}(-x)^k\right) = \frac{1}{2}\sum_{k=0}^{\infty} x^{2k}$. So, $f(x) = \sum_{k=0}^{\infty} kx^{2k-1}$. Letting k = 10, gives the answer.

2. B— By Taylor Expansion, $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$ At $x=1$, we get $\sin 1$.

3. E $\left(\frac{5}{16}\right)$ -- Let $S = \sum_{n=0}^{\infty}\frac{n}{5^n}$, then $\frac{1}{5}S = \sum_{n=0}^{\infty}\frac{n}{5^{n+1}} = \sum_{n=0}^{\infty}\frac{n+1}{5^{n+1}} - \sum_{n=0}^{\infty}\frac{1}{5^{n+1}} = \sum_{n=0}^{\infty}\frac{n}{5^n} - \frac{1/5}{1-1/5} = S - \frac{1}{4}$. So, $\frac{1}{5}S = S - \frac{1}{4} \Rightarrow \frac{4}{5}S = \frac{1}{4} \Rightarrow S = \frac{5}{16}$.

4. B—The partial sums of this sum are equal to

$$\left(\frac{1}{1}+\frac{1}{2}+\ldots+\frac{1}{3n}\right) - 3\left(\frac{1}{3\cdot 1}+\frac{1}{3\cdot 2}+\ldots+\frac{1}{3\cdot n}\right) = \frac{1}{n+1}+\frac{1}{n+2}+\ldots+\frac{1}{3n} = \frac{1}{n}\left(\frac{1}{1+\frac{1}{n}}+\frac{1}{1+\frac{2}{n}}+\ldots+\frac{1}{1+\frac{2n}{n}}\right).$$

This is a Riemann sum, so as n → ∞ the partial sums converge to $\int_0^2 \frac{1}{1+x}dx = \ln 3$.

5. A—The first three derivatives of g are determined by the first three derivatives of f. Additionally, $f(0) = g(0) = 0$. Let f and $g = f^{-1}(x)$ be new functions whose first three derivatives at zero equal those of f and g respectively. By Taylor Series expansion, we see that $f(x) = -\ln(1-x)$ is a good choice. Then, $g(x) = 1 - e^{-x}$ and $g^{(3)}(0) = g^{(3)}(0) = e^0 = 1$.

6. B—In order for a_n to have a limit L, it must be that $x^L = L$, so that $x = L^{1/L}$. Otherwise, we would be able to extend the recurrence and converge to a different limiting value. So, we want the max of the function $f(L) = L^{1/L}$. So, solve $\frac{df}{dL} = 0$. Since, $\frac{df}{dL} = \frac{d}{dL}e^{\frac{\ln L}{L}} = L^{1/L}\left(\frac{1}{L^2} - \frac{\ln L}{L^2}\right)$. Also, L = e. Thus, the max value for x is $f(e) = e^{1/e}$. (You can check 2nd derivative to ensure it is a max.)

7. D— $2^x = e^{x\ln 2}$ and $e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!}$. So, $2^x = e^{x\ln 2} = \sum_{n=0}^{\infty}\frac{(\ln 2)^n}{n!}x^n$.

8. C— $\lim_{n\to\infty} S_n = \lim_{n\to\infty}\frac{n}{n+1} = \lim_{n\to\infty}\frac{1}{1+\frac{1}{n}} = 1$

9. E—($-1\le x<5$) $\lim_{n\to\infty}\left|\dfrac{\dfrac{(x-2)^{n+1}}{(n+1)(3^{n+1})}}{\dfrac{(x-2)^n}{n(3^n)}}\right|=\left|\dfrac{x-2}{3}\right|<1$ or $-1<x<5$. Checking endpoints shows that using 5 gives a harmonic series, which is divergent and using -1 gives an alternating harmonic series, which is convergent. So, we include -1 in the answer.

10. D—The Taylor series for $\sin x$ about $x=\dfrac{\pi}{2}$ is $1-\dfrac{\left(x-\frac{\pi}{2}\right)^2}{2!}+\dfrac{\left(x-\frac{\pi}{2}\right)^4}{4!}-\dfrac{\left(x-\frac{\pi}{2}\right)^6}{6!}$. So, the coefficient is $\dfrac{-1}{6!}$ or $\dfrac{-1}{720}$.

11. C— $400=\dfrac{7}{2}(101+a_7)\Rightarrow a_7=156.14$.

12. D—Double the sum of squared whole #s from 0 to $n\Rightarrow 2\cdot\dfrac{n(n+1)(2n+1)}{6}=2\cdot\dfrac{(20)(21)(41)}{6}=5740$.

13. C—Sum of infinite series: $S=\dfrac{\sin\theta}{1-\sin\theta}=\dfrac{3}{2}\Rightarrow\sin\theta=\dfrac{3}{5}$.

14. C— $\sum_{i=1}^{20}\left(x^3\Big|_{i+1}^{i+3}\right)=\sum_{i=1}^{20}(i+3)^3-\sum_{i=1}^{20}(i+1)^3=(4^3+5^3+6^3+\ldots+23^3)-(2^3+3^3+4^3+\ldots+21^3)=$
$22^3+23^3-2^3-3^3=22,780$.

15. A—In general, $\binom{n}{0}^2+\ldots+\binom{n}{n}^2=\binom{2n}{n}$.

16. A— $\lim_{n\to\infty}\dfrac{\left(\sqrt{n+1}-\sqrt{n+2}\right)\left(\sqrt{n+1}+\sqrt{n+2}\right)}{\sqrt{n+1}+\sqrt{n+2}}=\lim_{n\to\infty}\dfrac{-1}{\sqrt{n+1}+\sqrt{n+2}}=0$

17. C— $\dfrac{a}{1-r}=8$ and $\dfrac{a^3}{1-r^3}=\dfrac{512}{7}\Rightarrow 7a^3=512(1-r)(1+r+r^2)$. Using the first equation, and subbing into this equation, then solving gives 2 solutions for *r*, either $r=\dfrac{1}{2}$ or $r=2$. If we use *r* =2, the series will not converge. Using *r* = ½, gives *a* = 4.

18. B—This series can be expressed as the sum of 2 infinite series:

$$\left(\frac{1}{7}+\frac{1}{7^3}+\ldots\right)+\left(\frac{2}{7^2}+\frac{2}{7^4}+\ldots\right)=\frac{\frac{1}{7}}{1-\left(\frac{1}{7}\right)^2}+\frac{\frac{2}{7}}{1-\left(\frac{2}{7}\right)^2}=\frac{3}{16}.$$

19. C—Remember that $\sum_{n=0}^{\infty}\frac{x^n}{n!}=e^x$. Substituting x = 2 gives $\sum_{n=0}^{\infty}\frac{2^n}{n!}$. So, $\sum_{n=0}^{\infty}\frac{2^{n-1}}{n!}=\frac{1}{2}e^2$.

20. D—The given info implies that the series is absolutely convergent, so I and II are true. If you move the negative sign on III in front of the sigma, then III reduces to I and it is also true, so all 3 are true.

21. D—If $\lim_{n\to\infty}a_n=L$, then $\lim_{n\to\infty}a_{n+1}=L$, so L must satisfy $L=\sqrt{4+3L}$ or $L^2=4+3L$. Solving this, gives $L=-1$ and $L=4$. Since L is positive, the only answer is 4.

22. D— Using the Ratio Test gives, $\lim_{n\to\infty}\frac{(n-1)!\left|x^{n+1}\right|/10^{n+1}}{n!\left|x^n\right|/10^n}=\lim_{n\to\infty}\frac{n+1}{10}\left|x\right|=\infty$, except for $x=0$. Hence, the radius of convergence is 0.

23. D—$\sum_{i=0}^{\infty}i(1+i)p^{i-2}=\sum_{i=0}^{\infty}i(1-i)p^{i-2}+2\sum_{i=0}^{\infty}ip^{i-2}$.

So, $\sum_{i=0}^{\infty}i(1-i)p^{i-2}=\frac{d^2}{dp^2}\sum_{i=o}^{\infty}p^i=\frac{d^2}{dp^2}\frac{1}{1-p}=\frac{d}{dp}\frac{1}{(1-p)^2}=\frac{2}{(1-p)^3}$.

Also, $\sum_{i=o}^{\infty}ip^{i-2}=\frac{1}{p}\sum_{i=o}^{\infty}ip^{i-1}=\frac{1}{p}\frac{d}{dp}\frac{1}{1-p}=\frac{1}{p(1-p)^2}$. Then, $\frac{2}{(1-p)^3}+\frac{2}{p(1-p)^2}=\frac{2}{p(1-p)^3}$.

24. C—$\sum_{k=n+1}^{2n}\frac{1}{k}=\frac{n}{n}\sum_{k=1}^{n}\frac{1}{k+n}=\sum_{k=1}^{n}\frac{1}{n}\frac{1}{1+\frac{k}{n}}$. This is a Riemann sum: $\int_1^2\frac{1}{x}dx=\ln 2$.

25. B—Distance = $\int_0^{\infty}\frac{1}{1+t^2}dt=\tan^{-1}t\,\Big|_0^{\infty}=\frac{\pi}{2}$.

26. C—Compute the Taylor expansion of $f^{//}(x)-2xf^{/}(x)-2f(x)$ to the 3rd term, which is $(2a+6bx+12cx^2+20dx^3+\ldots)-(2x+4ax^2+6bx^3+\ldots)-(2+2x+2ax^2+2bx^3+\ldots)$. All the coefficients should be 0, so $2a-2=0$, $6b-4=0$, $12c-6a=0$ and $20d-8b=0$. Solve the equations for answer.

27. A—In order to find the remainder mod 7, evaluate the sequence mod 7: $1 + 3 + 1 \equiv 5$ mod 7 and $5+3\cdot 1+1\equiv 2$ mod 7 and so on. The sequence repeats itself after 6 iterations, producing $1,1,1,5,2,4,1,1,1,\ldots$ Since $2013 \equiv 3$ mod 6, then $a_{2013} \equiv a_3 \equiv 5$ mod 7.

28. D—Separate the sum into 3 parts: one where $y = x$, one where $y = x + d$, and one where $y = x - d$ for some integer $d > 0$. The 1st is $S_1 = \sum_{x=0}^{\infty} \frac{1}{2^{2x}} = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$. The 2nd is $S_2 = \sum_{d=1}^{\infty}\sum_{x=0}^{\infty} \frac{1}{2^{2x+2d}} = \sum_{d=1}^{\infty} \frac{1}{2^{2d}} \sum_{x=0}^{\infty} \frac{1}{2^{2x}} = \frac{4}{3}\sum_{d=1}^{\infty} \frac{1}{2^{2d}} = \frac{4}{9}$. The 3rd sum, S_3, is just obtained by switching x and y, so $S_3 = S_2$. Then the total sum $S = S_1 + 2S_2 = \frac{20}{9}$.

29. A—There are 9 numbers with an arithmetic sequence of difference 0 (1111 through 9999). There are 6 with an arithmetic sequence of difference 1 (1234 through 6789). There are 3 with an arithmetic sequence of difference 2 (1357 through 3579). There are 7 with an arithmetic sequence of difference −1 (3210 through 9876). There are 4 with an arithmetic sequence of difference −2 (6420 through 9753), and there is 1 with a difference of −3 (9630). The answer is therefore 9 + 6 + 3 + 7 + 4 + 1 =30.

30. B—The orderings of the vowels and consonants are independent of how the vowels and consonants are ordered relative to each other. So first we compute the number of ways to arrange the vowels and consonants. The 4 V's partition the 7 C's into 5 possibly empty blocks. Since the maximum these blocks can have to satisfy the rules are 1,2,2,2, and 1, exactly one of them must be one short. There are 5 ways to do this. Then we multiply the number of permutations of the vowels AEAI and consonants MTHMTCS, of which there are $\frac{7!}{4}\cdot\frac{4!}{2} \Rightarrow 15\cdot 7!$.