For the purposes of this test, let ϕ represent the golden ratio, and the first two terms of the Fibonacci sequence are $F_0 = 0$ and $F_1 = 1$. Remember $\phi^2 = \phi + 1$. May the golden odds be ever in your favor!

The first 5 questions will pertain to the history of ϕ ...

1. What mathematician gave the first recorded definition of the golden ratio, referring to it (translated into English) as the "extreme and mean ratio"?

   A) Euler B) Euclid C) Fibonacci D) Pythagoras E) NOTA

2. What was the name of Luca Pacioli's illustrative work on the golden ratio?

   A) *De Aure Proportione* B) *De Magne Proportione* C) *De Divina Proportione*
   D) *De Mysterie Proportione* E) NOTA

3. In honor of what Greek sculptor was phi (ϕ), the symbol of the golden ratio, chosen?

   A) Phidias B) Phidippides C) Phiten D) Phirrhus E) NOTA

4. Who first proved that the golden ratio is the limit of the ratio of consecutive Fibonacci numbers?

   A) Kepler B) Pacioli C) Lucas D) Binet E) NOTA

5. Who is credited for the formula that determines the n^th Fibonacci number (F_n):

$$F_n = \frac{\left(1+\sqrt{5}\right)^n - \left(1-\sqrt{5}\right)^n}{2^n\sqrt{5}} = \frac{\phi^n - \left(-\phi\right)^{-n}}{\sqrt{5}}$$

   A) Bonnet B) Lucas C) Cauchy D) Binet E) NOTA

And now onto the good stuff!

6. The golden ratio is:
   A) 1.61803
   B) The ratio of two parts of a line such that the longer part divided by the smaller part is also equal to the whole length divided by the longer part
   C) The ratio of two successive numbers of the Fibonacci sequence
   D) The ratio of the width of the Parthenon to its height, viewed from the front
   E) NOTA

7. Which of the following is false?
   A) The golden ratio is an irrational number
   B) The golden ratio is a transcendental number
   C) The golden ratio is a complex number
   D) The golden ratio is a real number
   E) NOTA

8. Calculate the 20th Fibonacci number.

A) 4181 B) 6435 C) 6765 D) 10,616 E) NOTA

9. A particular circle has a radius of f . What is the circle's enclosed area? (*Hint:* $f^n = f^{n-1} + f^{n-2}$)

A) $p\left(f^5 - f^3\right)$ B) $2pf$ C) $\pi\left(1+\frac{1}{\phi}\right)$ D) $\pi\left(3-\frac{1}{\phi^2}\right)$ E) NOTA

10. A golden rectangle has a shorter side of 32 meters. What is the longer side in meters, approximating $\sqrt{5}$ » $9/4$?

A) 49 B) 50 C) 51 D) 52 E) NOTA

11. Simplify: $\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\ddots}}}$

A) $\frac{1}{f}$ B) $\frac{f}{2}$ C) f D) f^2 E) NOTA

12. Find the volume of the tetrahedron with vertices $(1,1,2)$, $(3,5,8)$, $(13,21,34)$, and $(55,89,144)$.

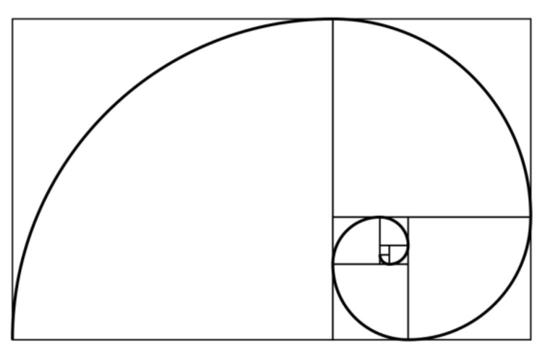
A) 89 B) 125 C) $\frac{144}{f}$ D) $72\sqrt{2}$ E) NOTA

13. Solve for x:

$$\phi^{\left(\phi^2-\frac{x-1}{\phi}\right)}-\frac{1}{\phi}=x$$

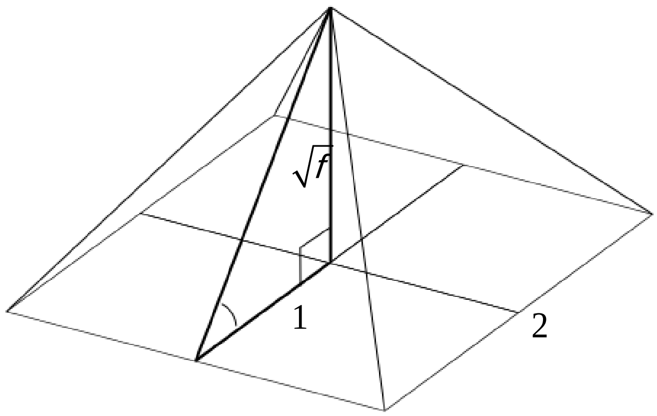
A) 0 B) 1 C) 2 D) $\sqrt{5}$ E) NOTA

14. The below depiction is a Fibonacci spiral, an approximation for the golden spiral. It is formed by drawing a smooth curve through opposite corners of adjacent squares. Find the side length of the largest square in this diagram, if the two smallest squares each have side length 1234.



A) 25,914
B) 30,386
C) 41,956
D) 67,870
E) NOTA

15. The Great Pyramid of Giza was built with dimensions roughly proportional to the golden ratio, with a height of 481 ft. and base dimension of 756 ft. You decide to build a scale model: a square pyramid with base dimension 2 ft. and height $\sqrt{f}$ ft. Determine the surface area to volume ratio of your scale model in 1/ft.



A) $3\left(\phi^{0.5}-\phi^{-0.5}\right)$ B) $3\sqrt{f}$ C) $3\left(\phi^{0.5}+\phi^{-0.5}\right)$ D) $6\sqrt{f}$ E) NOTA

16. Fiveonacci is trying to express the golden ratio only using the number 5, decimals, and order of operations operators. He comes up with the following representations:

i. $f = .5 + .5\,´\,5^{.5}$

ii. $f = \dfrac{5+5^{.5}}{5 - .5}$

iii. $\phi = \left(\dfrac{5+5^{.5}}{5-5^{.5}}\right)^{.5}$

Which of Fiveonacci's representations are true?

A) i. only B) i., ii. only C) i., iii. only D) i., ii., iii. E) NOTA

17. The Fibonacci sequence (n[th] term F_n) can be extended for to a negative index n by using the following relation $F_{n-2} = F_n - F_{n-1}$ for all n. Determine the 8[th] "negafibonacci" number, F_{-8}.

Hint:

F_{-8}	F_{-7}	F_{-6}	F_{-5}	F_{-4}	F_{-3}	F_{-2}	F_{-1}	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8
								0	1	1						

A) 21 B) -21 C) 13 D) -13 E) NOTA

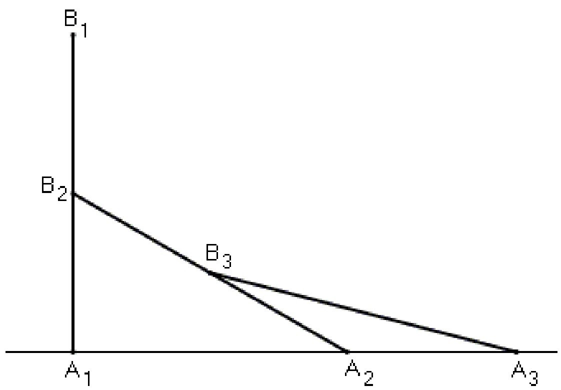
18. Mr. Recursionson is interested in forming sequences with 1's and 2's that sum to a specific number. He observes that there are 3 sequences of 1's and 2's sum to 3:

$$f(3)=1+1+1=1+2=2+1$$

How many sequences of 1's and 2's sum to 14? (Hint: Mr. Recursionson's name is significant in the context of this problem… look for a pattern!)

A) 105 B) 377 C) 452 D) 610 E) NOTA

19. Three equal segments A_1B_1, A_2B_2, A_3B_3, each of length 2, are positioned in such a way that endpoints B_2, B_3 are the midpoints of A_1B_1, A_2B_2 respectively, while the endpoints A_1, A_2, A_3 are on a line perpendicular to A_1B_1. Determine A_1A_3.



A) $\phi\sqrt{2}$ B) $\phi\sqrt{3}$ C) $\phi\sqrt{5}$ D) 2ϕ E) NOTA

20. Simplify, where F_n is the n^th Fibonacci number and a is an integer ≥ 1: $\lim_{n\to\infty}\frac{F_{n+a}}{F_n}$

A) $F_a\phi+F_{a-1}$ B) ϕ^{a-1} C) $(a-1)\phi+(a-2)$ D) $F_{a-1}\phi+F_{a-2}$ E) NOTA

21. Below is the generating function for the Fibonacci sequence:

$$f(x)=\sum_{n=0}^{\infty}F_nx^n=\frac{x}{1-(x+x^2)}=x+x^2+2x^3+3x^4+\cdots$$

Solve for n, where F_n is the n^th Fibonacci number: $f\left(\frac{1}{10}\right)=\frac{10}{F_n}$

A) 89 B) 10 C) 55 D)11 E) NOTA

22. Lucas numbers, L_n, are a slight variation on Fibonacci numbers, F_n, defined as follows:

$$L_n = \begin{cases} 2 & \text{if } n = 0; \\ 1 & \text{if } n = 1; \\ L_{n-1} + L_{n-2} & \text{if } n > 1. \end{cases} \qquad F_n = \begin{cases} 0 & \text{if } n = 0; \\ 1 & \text{if } n = 1; \\ F_{n-1} + F_{n-2} & \text{if } n > 1. \end{cases}$$

Lucas numbers relate to Fibonacci numbers by the following expression: $\phi = \sqrt[n]{\dfrac{L_n + F_n\sqrt{5}}{2}}$

Using the above information, determine which of the following are true:

i. $\sqrt{\dfrac{7+3\sqrt{5}}{2}} - 1 = \phi$

ii. $47 + 21\sqrt{5} = 2f^7$

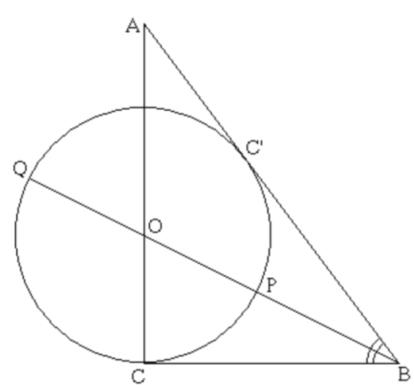
iii. $\sqrt[13]{\dfrac{521+233\sqrt{5}}{2}} = \sqrt[5]{\dfrac{11+5\sqrt{5}}{2}}$

A) i. only B) ii. only C) i.,ii. only D) i.,iii. only E) NOTA

23. A triangle has sides in geometric progression. What is the length of the range of the possible common ratios of the progression?

A) $\dfrac{1}{f}$ B) 1 C) f D) $2f - 1$ E) NOTA

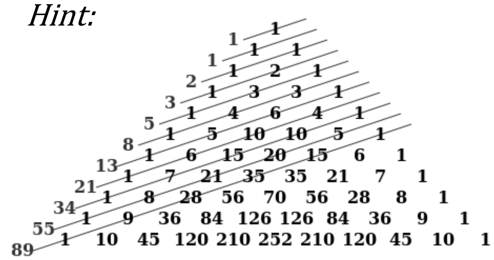
24. Let ABC be such a triangle with BC = 3, AC = 4 and AB = 5. Let O be the foot of the angle bisector at B. Draw a circle with center O and radius CO. Extend BO to meet the circle at Q and let P be the other point of intersection of BO with the circle. Determine the relationship PQ/BP.



A) $\dfrac{3}{2}$ B) $\dfrac{5}{3}$ C) $\dfrac{8}{5}$ D) *Not enough information given* E) NOTA

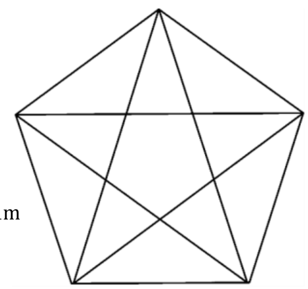
25. Evaluate $f(10)$: $f(n)=\sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \binom{n-k-1}{k}$

Hint:

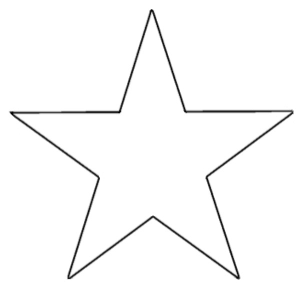


A) 55
B) 70
C) 89
D) 252
E) NOTA

26. Heinrich Agrippa plans on building a golden flower garden. He constructs a fenced enclosure in the shape of a regular pentagon with side length of 1 meter. He wishes to plant his golden flowers within the fencing in the configuration of a regular pentagram (5-pointed star constructed with 5 straight lines), shown left. However, realizing that he doesn't have as many flowers on hand as expected, Heinrich only end up planting along the exterior of the pentagram, shown right.



Planned Flower Arrangement
(outer pentagon not included)


Planted Flower Arrangement

How many meters of flowers does Heinrich ultimately plant? (Hint: $\cos(3p/5)=\frac{1}{4}(1- \sqrt{5})$)

A) $\frac{10}{f}$ B) $5f$ C) $5\left(\phi+\frac{1}{\phi}\right)$ D) $5+f$ E) NOTA

The next three questions pertain to the origins of the Fibonacci sequence…

27. The original problem that Fibonacci investigated, in the year 1202, was how quickly bunnies could breed. In his thought experiment, each pair of child bunnies matures into a pair of adult bunnies in one timestep (produces no offspring), and each pair of adult bunnies produces one pair of child bunnies on each timestep. Let A_t and C_t represent the number of pairs of adult and child bunnies at time t, respectively. If $A_0 = 8$ and $C_0 = 5$, calculate A_6.

A) 55 B) 89 C) 144 D) 233 E) NOTA

28. In Fibonacci's same thought experiment, he adds N new pairs of adult bunnies to the population at each timestep. Derive an expression for A_t in terms of A and N to model the number of adult bunnies in the population at time t, assuming A_0 and C_0 are unknown.

A) $A_{t-1}+A_{t-2}+N$ B) $A_{t-1}+A_{t-2}+t´ N$ C) $A_{t-1}+A_{t-2}+(t-1)´ N$
D) $A_{t-1}+A_{t-2}+2N$ E) NOTA

29. Fibonacci's lesser-known (and hypothetical) cousin Fabonucci was concerned about the exploding bunny populations in his cousin's model. He started contemplating the benefits of introducing snakes into the bunnies' home to limit population growth.

He repeated Fibonacci's original experiment (see question 27) with several species of pythons:

- Snakes of the *P. miniscułus* species can only eat child bunnies. They eat a proportion $1-a$ of each generation of young bunnies before they can mature into adults.
- Snakes of the *P. voluminus* species only bother eating adult bunnies. They eat a proportion $1-a$ of each generation of adult bunnies, but only *after* they've had a chance to procreate.
- Snakes of the *P. monstruous species* only bother eating adult bunnies. They eat a proportion $1-a$ of each generation of adult bunnies, *before* they have a chance to procreate.
- Snakes of the *P. eatsalottus* species eat a proportion $1-a$ of both child and adult bunnies, and get to the adults before they can procreate.

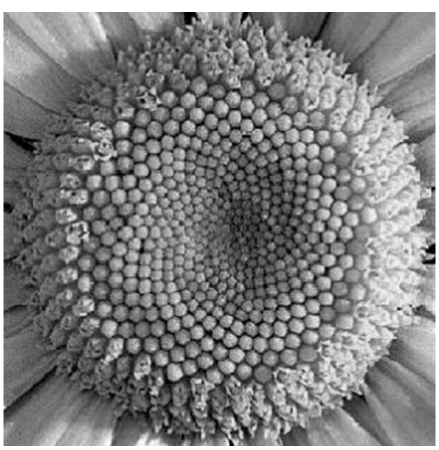
In a particular experiment, Fabonucci observes the following growth model:

$$A_t = aA_{t-1} + aA_{t-2}$$

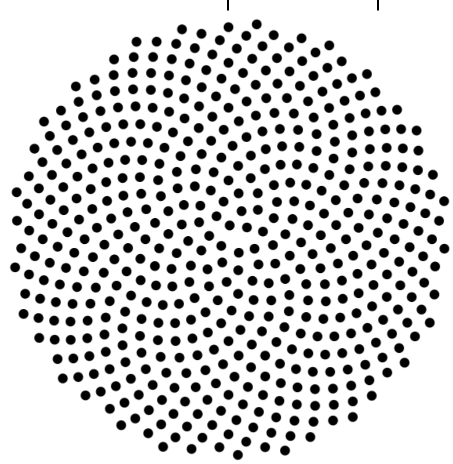
What species of snake did Fabonucci introduce into the bunny population?

A) *P. miniscułus* B) *P. voluminus* C) *P. monstruous* D) *P. eatsalottus* E) NOTA

30. Nature displays countless applications of the golden ratio, suggesting its importance as a fundamental characteristic of the universe. Fibonacci numbers dictate the natural structure of pinecones, shells, fingers, faces, hurricanes, spiral galaxies, and more. In this particular instance, the seeds within the center of a sunflower (shown left) are arranged in bidirectional Fibonacci spirals. Using the simplified diagram on the right, count the spirals both clockwise and counter clockwise, and let these quantities be a and b respectively. Calculate $\begin{vmatrix} b & a \\ a & a+b \end{vmatrix}$.



Seeds in Center of Sunflower



Spirals in Center of Sunflower

A) -1 B) 1 C) 1429 D) -1429 E) NOTA