- *NOTA means None of These Answers*
- *"Odds" means odds against*
- *If I bet with 5:2 odds, a bet of $1 will receive a $2.50 payout plus the original $1*
- *"Even odds" means the odds are 1:1*
- *A "fair" game has an expected profit of $0*

1. You enter a raffle for $100,000 with a $100 dollar payment. If there are 1000 tickets in the raffle (including yours), which of the following is closest to the expected value?

   A. Positive B. Negative C. Break Even ($0) D. Not enough info E. NOTA

2. Fill in the blanks: Games can be classified by their source(s) of uncertainty. Using this method, chess would be best classified as a _____ game, and rock-paper-scissors would be best classified as a _____ game.

   A. strategic, strategic B. strategic, chance
   C. combinatorial, chance D. combinatorial, strategic E. NOTA

3. From a standard deck of cards, what is the probability I draw a pair (two cards of the same value) if I draw two cards without replacement?

   A. $\frac{1}{17}$ B. $\frac{1}{51}$ C. $\frac{1}{102}$ D. $\frac{2}{51}$ E. NOTA

4. Computer AIs use algorithms to determine their moves in games such as chess. These algorithms involve searching the game trees. Relative to the game of Go, Chess has a lower game complexity and difficulty of evaluating the board. Because of its complexity, Go AI programs make notable use of what algorithm not found in many game AIs to reduce computation time?

   A. Monte Carlo Tree Search B. Minimax
   C. Linear Tree Search D. Binary Tree Search E. NOTA

5. I bet with even odds that there will be at least one 6 in n rolls of a standard six-sided dice. What is the smallest value of n for which I am expected to profit?

   A. 3 B. 4 C. 5 D. 6 E. NOTA

Use the following information for questions 6-7:
Katie and Rob play a game where each has an equal chance of scoring a point, but they are interrupted mid-game. Katie and Rob have 8 and 7 points respectively when they have to stop.

6. If you need 9 points to win and they decide to split the stakes fairly upon stopping, what fraction should Katie receive?

   A. 1/2 B. 5/8 C. 3/4 D. 7/8 E. NOTA

7. If you need 10 points to win and they decide to split the stakes fairly upon stopping, what fraction should Rob receive?

   A. 1/4 B. 5/16 C. 3/8 D. 2/5 E. NOTA

8. It is well known that when rolling two standard six-sided die, I am most likely to roll a seven (the sum of the two rolls). I am half as likely to roll a sum of *n*. What is the smallest value of *n*?

   A. 3 B. 4 C. 5 D. 6 E. NOTA

9. A dealer begins to deal out a 52 card deck starting with player 1. If you are player 3, what is the probability the first card you are dealt is a spade if you do not know the cards dealt to players 1 or 2?

   A. 1/4 B. 13/50 C. 6/25 D. 11/50 E. NOTA

10. Probability theory traces its roots to correspondence between which two mathematicians?

    A. Fermat, Euler B. Pascal, Descartes
    C. Pascal, Euler D. Fermat, Descartes E. NOTA

11. You challenge your friend to a game. Your friend rolls a single six-sided die. If he rolls a six, you'll give him $12. If he rolls an odd number, you'll give him $6. If he rolls something else, you give him nothing. If the game is fair, what should you charge per roll?

    A. $5 B. $6 C. $7 D. $8 E. NOTA

12. Assume the registration fee for the Mu Alpha Theta National Convention is $500 before April 1st and $650 after. What value should the probability you attend be **greater than** so that it makes sense to register early (expected net gain by registering early)?

A. 1/2 B. 13/23 C. 7/10 D. 10/13 E. NOTA

13. Three moves into a chess game between grandmaster Gary Kasparov and IBM's Deep Blue in 1997, Kasparov achieved a board position that had occurred only once in master-level competition. This circumstance would've most drastically affected which of Deep Blue's functions?

A. Minimax optimization B. Database searches
C. Game tree search depth D. Use of heuristics E. NOTA

14. Consider the St. Petersburg Paradox: You flip a coin until it comes up on heads. If it comes up on heads on the n^{th} flip, you get $\$2^n$. Now assume that after N games, if heads has not come up, you are cut off, ending the game, and paid $\$2^N$. What is the expected value of this game (in dollars)?

A. N B. $2N$ C. N^2 D. 2^N E. NOTA

15. You flip a coin until it comes up on heads. You bet \$2 with even odds that the first flip will be heads, \$4 that the second flip will be heads (if it didn't turn up on the first), etc. What is your expected profit?

A. \$0 B. \$2 C. $\$2^n - 2$ D. $\$\infty$ E. NOTA

16. Consider the game Nonsense with two players, A and B. A first chooses one of three triangles, labeled 1, 2, and 3 respectively. B then chooses one of the two remaining triangles and A must take the last one. The payoff to each player is the sum of their triangles' numbers. Now many nodes are on the game tree for Nonsense? The game tree must include all moves made.

A. 9 B. 10 C. 15 D. 16 E. NOTA

17. Your math club has a fundraiser at the school carnival. For a payment of \$1, you let a player choose a number and then roll three dice. If the player's number comes up once, they win \$2, if it comes up twice they win \$3, and if it comes up three times they win \$4. If the game is played 216 times, what is your club's expected profit to the nearest dollar?

A. \$0 B. \$17 C. \$34 D. \$51 E. NOTA

18. When using binomial probabilities, once the number of trials *n* is great enough, which of the following is the most specific type of curve by which a frequency chart of outcomes is approximated?

A. Exponential B. Logarithmic C. Normal curve D. S-curve E. NOTA

19. Fill in the blanks: As the number of repeated plays of a house-favoring game _____, the probability of winning decreases. This decrease is _____ dramatic for games with a larger house favor.

    A. increases, more B. increases, less C. decreases, more D. decreases, less E. NOTA

20. Rob and Tyler play Nim with two piles of chips: one with 100 and one with 200. Tyler starts the game and they alternate turns. In each turn each of the players chooses one of the piles and takes any number of chips from the pile. The player to take the last chip wins. If Tyler is playing optimally, how should he start?

    A. 100 chips from the 200 pile B. 199 chips from the 200 pile
    C. 99 chips from the 100 pile D. 50 chips from the 100 pile E. NOTA

21. Assume that when a twenty-one deck is "unfavorable" your probability of winning a hand is 2/5, and when it is favorable, your probability of winning a hand is 2/3. The deck is favorable with probability ¼ and unfavorable with probability ¾. If you bet $1 whenever it is unfavorable and $5 when it is favorable, what is the expected profit per play in dollars? Bets are with even odds.

    A. 39/30 B. 5/12 C. 23/60 D. 4/15 E. NOTA

22. Dr. Morris and Nick play a game where they count together to 20. Each can choose to say one number or two numbers in the count (ex. 1 or 1,2), and the next person resumes the count where the other left off. Whoever says 20 wins. If Dr. Morris starts and both players play optimally, who will win?

    A. Nick B. Cannot be determined until turn two
    C. Cannot be determined until turn three D. Dr. Morris E. NOTA

23. Another game is played by Dr. Morris and Adam with the same rules as the above game (#22) but with a count to 50 and the ability to say up to seven numbers at the time. Adam starts and counts six numbers. If playing optimally, how many numbers will Dr. Morris count on his first turn?

    A. 1 B. 2 C. 3 D.4 E. NOTA

24. I play a game with my brother. We take turns rolling a single die and the first person to roll a six wins. If he goes first, what's the probability I win?

    A. 5/36 B. 1/6 C. 5/11 D. 1/2 E. NOTA

25. The probability of hitting a circular dart board with radius R a distance R from the center is directly proportional to the radius cubed. If you have a 25% chance of hitting within one unit of the center, what is the radius of the dart board?

A. $\sqrt[4]{2}$ B. $\sqrt{2}$ C. $\sqrt[4]{8}$ D. 2 E. NOTA

26. In the game of Nim, two players take turns removing chips from a number of piles. Zach and Will are playing with four piles that have 11, 12, 13, and 14 chips in them respectively. If Will goes first and plays optimally, how many chips should he remove from the pile of 14?

A. 2 B. 4 C. 7 D.14 E. NOTA

27. How many of the following attributes could NOT describe a combinatorial game?

i. Imperfect information
ii. No chance
iii. Turn-based
iv. Two players
v. Three players
vi. Solved game

A. 0 B. 1 C. 2 D. 3 E. NOTA

28. Consider the below zero-sum game in normal form, where the numbers represent the payout for A. Which strategy will A use?

	B1	B2	B3
A1	3	1	-6
A2	-3	3	-8
A3	-4	5	-7
A4	2	-10	-7

A. A1 B. A2 C. A3 D. A4 E. NOTA

29. A specification of strategies is in Nash Equilibrium if no player can benefit by unilaterally changing his or her strategy (with all others staying fixed). How many points of pure strategy Nash Equilibrium are there in the following game?

	B1	B2
A1	1 1	10 -10
A2	-10 10	-30 -30

A. 0 B. 1 C. 2 D. 3 E. NOTA

30. In rock paper scissors, assuming both players choose a move at random, what is the probability of a draw?

A. 1/9 B. 1/3 C. 1/2 D. 2/3 E. NOTA