GAMES Solutions
MAO National Convention 2015

Answers:

| | | | | |
|---|---|---|---|---|
| 1. **C** | 7. **B** | 13. **B** | 19. **A** | 25. **B** |
| 2. **D** | 8. **B** | 14. **E** | 20. **A** | 26. **B** |
| 3. **A** | 9. **A** | 15. **B** | 21. **D** | 27. **C** |
| 4. **A** | 10. **E** | 16. **D** | 22. **D** | 28. **A** |
| 5. **B** | 11. **A** | 17. **B** | 23. **D** | 29. **C** |
| 6. **C** | 12. **D** | 18. **C** | 24. **C** | 30. **B** |

Solutions:

1. The expected value of this game is $100{,}000\left(\frac{1}{1000}\right) + 0\left(\frac{999}{1000}\right) = 100$. We subtract the amount paid from this to get the expected profit: $100 - 100 = 0$. Therefore, the answer is **C**.

2. In chess, uncertainty comes from the sheer number of possible moves, as both players have the same (and perfect) information. Therefore, it is a combinatorial game. Rock-paper-scissors, on the other hand, is not turn based and has no source of chance such as dice or shuffling cards. However, because turns are taken simultaneously, each player has imperfect information about what the other is doing. Therefore, it is a strategic game. **D**

3. It doesn't matter what the first card drawn is. Afterwards, there are three of the same number in the deck which has 51 cards left. $\frac{3}{51} = \frac{1}{17}$ **A**

4. Because of the size of the Go game tree, the Monte Carlo Tree Search is used which takes random samplings of the game tree to reduce computation time. Note that Minimax, while also used, is commonly used in other game AIs **A**

5. Since we are betting with even odds, we make a profit when the true probability is greater than $1/2$ . The probability of there being at least one six is 1 minus the probability of no six in n rolls. We solve: $1 - \frac{5^n}{6^n} > \frac{1}{2} \;\Rightarrow\; 6^n - 5^n > \frac{6^n}{2} \;\Rightarrow\; 6^n > 2(5^n)$. Trying values of n, we see that 4 is the smallest that works. **B**

6. They can fairly split the stakes according to the probability each has of winning. Katie has a $\frac{1}{2}$ chance of winning on the next turn. If Rob gets the point, she has a $\frac{1}{2} * \frac{1}{2} = \frac{1}{4}$ chance of winning on the subsequent turn. If Rob gets the next two points, he wins. Therefore, Katie's probability of winning is $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$, and that is the fraction of the stakes she should receive. **C**

7. We can construct a chart with the possible outcomes and probabilities of each:

| | Turn 1 | Turn 2 | Turn 3 | Turn 4 | Winner | Probability |
|---|---|---|---|---|---|---|
| Outcome 1 | K | K | - | - | K | 1/4 |
| 2 | K | R | K | - | K | 1/8 |
| 3 | K | R | R | K | K | 1/16 |
| 4 | K | R | R | R | R | 1/16 |
| 5 | R | R | R | - | R | 1/8 |
| 6 | R | R | K | K | K | 1/16 |
| 7 | R | R | K | R | R | 1/16 |
| 8 | R | K | K | - | K | 1/8 |
| 9 | R | K | R | R | R | 1/16 |
| 10 | R | K | R | K | K | 1/16 |

From our chart, we see Rob has a $\frac{1}{16}+\frac{1}{8}+\frac{1}{16}+\frac{1}{16}=\frac{5}{16}$ chance of winning. **B**

8. There are 6 ways to get a 7, and there are 36 total combinations of two numbers that can be made with two die, giving a probability of $\frac{6}{36}$. There are 3 ways of rolling a four (1,3; 3,1; 2,2). This gives a probability of $\frac{3}{36}$ which is half of that of 7. **B**

9. The cards dealt to players 1 and 2 have not been exposed to you. Therefore, the probability is still 1/4. **A**

10. Probability theory traces its beginnings to a series of letters between Blaise Pascal and Pierre de Fermat. **E**

11. If the game is fair, it should have an expected profit of 0, meaning the expected value equals the amount paid. The expected value is $\frac{1}{6}(12)+\frac{1}{2}(6)+\frac{1}{3}(0)=5$. Therefore, you should charge \$5. **A**

12. It makes sense to register early when the expected gain is greater than 0. In other words, the expected value of saving \$150 is greater than the expected value of losing \$500. Mathematically, this is $150p-500(1-p)>0 \rightarrow \ 650p>500 \rightarrow \ p>10/13$. **D**

13. Deep Blue used databases of past games to evaluate moves. Achieving a position only seen once before would had all but obliterated the advantage of looking at these past games. **B**

14. $EV=\sum_{i=1}^{N} 2^i\left(\frac{1}{2^i}\right)+2^N\left(\frac{1}{2^N}\right)=N+1$. **E**

15. If you win on the first flip, you will get \$2. If you win on the second, you will have lost \$2 but won \$4 for a net of \$2. This pattern continues for all flips as n approaches infinity. Therefore, the expected profit is \$2. **B**

16. Draw out the game tree. The starting position has three options (3 triangles), which each have 2 options (two triangles left), which each have 1 option (one triangle remaining). Count up the nodes: 1 + 3 + 3(2) + 6(1) = 16. **D**

17. Expected value per play for the club: $EV = 1 - 2\left(\frac{3*1*5*5}{6^3}\right) - 3\left(\frac{3*1*1*5}{6^3}\right) - 4\left(\frac{1}{6^3}\right) = \frac{17}{216}$. Playing 216 times would yield an expected profit of $216\left(\frac{17}{216}\right)$ = \$17. **B**

18. The frequency chart resembles a hump shape. The only curve with this shape listed is the normal curve. **C**

19. The probability of walking away with a winning decreases as the number of plays <u>increases</u>, as your winnings (or lack thereof) align more with the expected value of the game. Clearly, the decrease is <u>more</u> dramatic for games with a larger house edge. **A**

20. Let a position in which the two piles are equal be called a P-position and one in which they are not be called an N-position. Notice that the game ends in a P-position, and from a P-position you must move to an N-position and vice versa. Therefore, Tyler wants to start by putting the game in a P-position, because it will then always be in a P-position as a result of his turn, eventually allowing him to win. To make the piles equal, he should take 100 from the 200 pile. **A**

21. $EV = \frac{1}{4}\left(\frac{2}{3}\right)(5) + \frac{1}{4}\left(\frac{1}{3}\right)(-5) + \frac{3}{4}\left(\frac{2}{5}\right)(1) + \frac{3}{4}\left(\frac{3}{5}\right)(-1) = 4/15$. **D**

22. Notice that no matter what Nick says, Dr. Morris can cause the last number he says to have the same value mod 3 as the last number he said during his previous turn. $20 \equiv 2 \; mod \; 3$, so Dr. Morris can win by counting two numbers on the first turn. **D**

23. Notice that now, Dr. Morris can cause the last number he says to have the same value mod 8 as the last number he said on the previous turn. $50 \equiv 2 \; mod \; 8$, so Dr. Morris wants to count 4 numbers to reach 10 on his first turn. **D**

24. The probability I win on the first turn is the probability he doesn't roll a six times the probability I do: $\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)$. The probability I win on the second turn is $\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)$. This continues, and we can sum these probabilities as an infinite geometric series:
$S = \frac{a}{1-r} = \frac{\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)}{1-\left(\frac{5}{6}\right)^2} = 5/11$. **C**

25. We have $P = kr^3$ for all r. We find probability of hitting within one unit of the center by integrating the function from 0 to 1: $\int_0^1 kr^3 dr = \frac{k}{4} = \frac{1}{4} \Longrightarrow k = 1$. We know that probabilities add to one, so we have $1 = \int_0^R r^3 dr = \frac{R^4}{4} \Longrightarrow R^4 = 4 \Longrightarrow R = \sqrt{2}$. **B**

26. A P-position (refer to the solution for #20) in the game of Nim is one in which the bitwise sum of the stacks' binary representation is 0. This is also known as the Nim sum. The binary representations of 11, 12, 13, and 14 are $1011_2$, $1100_2$, $1101_2$, and $1110_2$ respectively. Taking the bitwise sum of 11, 12, and 13 we have $1010_2$. Therefore, we must take 4 away from the stack of 14 to get a stack of $1010_2 = 10$ so that the bitwise sum is 0. **B**

27. Combinatorial games must have perfect information, have no chance, be turn-based, and have two players. Some are solved, yet some remain unsolved (i.e. chess). Therefore, i and v cannot describe any combinatorial game. **C**

28. No matter what strategy B uses, A4 is clearly inferior to A1, so remove that row. Seeing that A won't use A4, B can tell that B2 is inferior to B3. Now it is clear A2 and A3 are also inferior to A1, so remove those as well. Therefore, A will use A1 and B will respond with B3.

29. Examining the board and testing each space with the definition, we see that the top right and bottom left corners are points of Nash Equilibrium. **C**

30. Your opponent has a one choice out of three that matches what you picked. The probability he or she does so is, therefore, 1/3. **B**