Assume numbers in all questions are expressed in base 10 unless otherwise specified. When applicable, NOTA stands for "None of these answers."

1) How many positive integral divisors does 2015 have?

   A) 4 B) 6 C) 8 D) 12 E) NOTA

2) What is the product of all the *positive and negative* integral divisors of 314?

   A) 314^2 B) 314^3 C) 314^4 D) 314^8 E) NOTA

3) How many digits are there in the repeating block of digits in the decimal expression of $\frac{12}{13}$?

   A) 5 B) 6 C) 11 D) 13 E) NOTA

4) Mr. Macfarlane, a math teacher at PHUHS, owns an orange farm. He allows his math students the opportunity to pick oranges in his groves for extra credit. Because his final exam was so hard, every one of his 60 math students participated. In total, they picked 31,415 oranges. Sara K., a very ambitious student, picked more than any other student. What is the least number of oranges that Sara K. could have picked?

   A) 178 B) 523 C) 524 D) 525 E) NOTA

5) When the decimal number 271 is converted to base 4, what is the sum of the digits of the base four number, expressed in base ten?

   A) 7 B) 9 C) 11 D) 12 E) NOTA

6) Let Q be the smallest integer greater than 1,000 that is not prime and has only one factor in common with 30!. What is Q?

   A) 1009 B) 1021 C) 1147 D) 1273 E) NOTA

7) If both 11^2 and 3^3 are factors of the positive integer $a \cdot 4^3 \cdot 6^2 \cdot 13^{11}$, where a is a positive integer, then what is the least possible value of a?

   A) 3267 B) 363 C) 121 D) 33 E) NOTA

8) In base eleven, the letter A is the symbol for 10. What is $1A9_{11}$ expressed in base ten?

A) 240 B) 251 C) 269 D) 271 E) NOTA

9) Suppose x is an integer such that $(x^2 - x - 1)^{x+2} = 1$. How many possible values of x exist?

A) 1 B) 2 C) 3 D) 4 E) NOTA

10) Let p represent a prime number. The ordered triple $(p, p+6, p+12)$ is called a sexy prime triplet if $p+6$ and $p+12$ are prime but $p+18$ is composite. How many sexy prime triplets exist if $p < 35$?

A) 4 B) 3 C) 2 D) 1 E) NOTA

11) Consider statements (I) and (II) below and determine whether the information provided in them can answer the question stated below. Select the appropriate answer that correctly describes whether the statements can answer the question.

Question: If x, y are integers, is $(x-1) > y$?

Statements: (I) x is a positive multiple of y
(II) $\frac{x}{y} > 1$

A) Statement (I) alone is sufficient, but statement (II) alone is not sufficient to answer the question asked
B) Statement (II) alone is sufficient, but statement (I) alone is not sufficient to answer the question asked
C) Both statements (I) and (II) together are sufficient to answer the question asked, but neither statement alone is sufficient
D) Each statement alone is sufficient to answer the question asked
E) Statements (I) and (II) together are not sufficient to answer the question asked, and additional data specific to the problem are needed

12) Consider the following statement: *Every perfect square is either divisible by 4 or is congruent to* _____ .

Select the answer below that, when plugged into the blank above, makes the statement true.

A) 1 mod 5 B) 1 mod 3 C) 1 mod 4 D) 4 mod 7 E) NOTA

13) The set of odd integers is closed under which of the following operations?

(I) Multiplication (II) Addition (III) Division

A) I only B) II only C) I, II only D) II, III only E) NOTA

14) $\prod(n)$ equals the number of positive integers that are less than n and relatively prime to n. What is $\prod(36)$?

A) 10 B) 12 C) 13 D) 14 E) NOTA

15) Given a triangle with sides of measure $2m^2+1, 2m^2+2$, and $4m^2+1$, where m is an integer not equal to 0, then which of the following statements must be true?

Hint: The area of a triangle with sides $, B, C = \sqrt{S(S-A)(S-B)(S-C)}$*, where* $S = \frac{A+B+C}{2}$

A) The triangle is a right triangle
B) The triangle is an obtuse triangle
C) The perimeter of the triangle is an integer divisible by 6
D) The area of the triangle formed is an integral number of square units
E) NOTA

16) Mrs. Lindar's disco store sold 72 disco balls for$\$a67.9b$. What is $a+b$?

A) 3 B) 5 C) 12 D) 14 E) NOTA

17) A *perfect number* is a positive integer that is equal to the sum of its proper positive divisors. A *triangular number* counts the objects that can form an equilateral triangle. Which one of the following numbers is triangular and perfect?

A) 120 B) 78 C) 36 D) 28 E) NOTA

18) The number A, when expressed in base 7, is $0.222\ldots_7$. When expressed in base 10, A can be written as $0.ppp\ldots_{10}$. What is the value of p?

A) 3 B) 5 C) 6 D) 7 E) NOTA

19) How many solutions in positive integers (m, n) are there to the equation:

$$6m^3 + 51n = 2015$$

A) 1 B) 2 C) 5 D) 7 E) NOTA

20) Let q be a randomly chosen two-digit positive integer. What is the probability that the fraction $6/q$ is reducible?

A) 1/2 B) 2/3 C) 4/5 D) 5/6 E) NOTA

21) Find the largest integer that evenly divides $n^5 - 5n^3 + 4n$ for all integers n.

A) 24 B) 60 C) 120 D) 240 E) NOTA

22) RuPaul is touring a nation in which coins are issued in two amounts, ¥2 and ¥5, which are made of iron and copper, respectively. If RuPaul has ten iron coins and ten copper coins, how many different sums from ¥1 to ¥70, inclusive, can she make with a combination of his coins?

A) 69 B) 68 C) 66 D) 60 E) NOTA

23) Which of the following is NOT prime?

A) 1,556,551
B) 2,442,113
C) 3,893,983
D) 3,999,991
E) 9,999,991

24) Consider the polynomial: $f(x) = x^5 + x^4 + 4x^3 + 3x^2 + x + 1$. Which of the following statements is FALSE?

I. $f(x)$ has at least one imaginary root
II. $f(x)$ has at least one positive real root
III. $f(x)$ has at least one negative real root

A) II only
B) III only
C) I, II only
D) II, III only
E) NOTA

25) $\sum_{k=0}^{10} \binom{10}{k} =$

A) 1,025 B) 1,024 C) 513 D) 512 E) NOTA

26) Liam rolls two fair dice. Let D_1 and D_2 be the face values of the two rolls. What is the probability that D_1 and D_2 are relatively prime?

A) 5/6 B) 23/36 C) 5/9 D) 13/36 E) NOTA

27) For integers B and C, if $(B+2)(C+3)$ is even, then $4BC$ must be divisible by:

A) 4 B) 8 C) 9 D) 12 E) NOTA

28) If $22{,}023 - n$ is divisible by 11, and $0 \leq n \leq 11$, what is n?

A) 9 B) 5 C) 3 D) 1 E) NOTA

29) Consider statements (I) and (II) below and determine whether the information provided in them can answer the question stated below. Select the appropriate answer that correctly describes whether the statements can answer the question.

Question: x is a positive integer less than 20. What is the value of x?

Statements: (I) x is the sum of two consecutive integers
(II) x is the sum of five consecutive integers

A) Statement (I) alone is sufficient, but statement (II) alone is not sufficient to answer the question asked
B) Statement (II) alone is sufficient, but statement (I) alone is not sufficient to answer the question asked
C) Both statements (I) and (II) together are sufficient to answer the question asked, but neither statement alone is sufficient
D) Each statement alone is sufficient to answer the question asked
E) Statements (I) and (II) together are not sufficient to answer the question asked, and additional data specific to the problem are needed

30) Evaluate the following: $2014^2 + 2015 - 2015^2 + 2014$

A) -4,030 B) 0 C) 2015 D) 4,028 E) NOTA