Where applicable, "E) NOTA" indicates that none of the above answers is correct.

Unless otherwise specified, a "die" or "dice" are fair and six-sided, numbered 1 through 6; a "coin" or "coins" are fair with a heads on one side and a tails on the other, and the result(s) of a "coin flip" or "coin flips" is/are either heads or tails (with equal probability) on each flip; and a "deck of cards" is a standard, 52-card deck with a deuce through ace each of clubs, diamonds, hearts, and spades—an ace may be counted as a high or low card in a poker hand, but not simultaneously both. Additionally, for playing cards, "rank" refers to the value of the card (2, 3, …, K, A).

1. How many distinct sequences of coin flips could result from four flips of a coin?

A) 4 B) 8 C) 12 D) 16 E) NOTA

2. Two cards are drawn from a deck of cards. If the first drawn card is revealed to be the Ace of Spades, what is the probability that the second drawn card is an ace?

A) $1/13$ B) $1/17$ C) $3/52$ D) $4/51$ E) NOTA

3. If a die is to be rolled repeatedly until a 5 or 6 is rolled, what is the expected number of rolls needed?

A) 1 B) 2 C) 3 D) 6 E) NOTA

4. From the complete list of positive integral factors of 600, two numbers are chosen. What is the probability that the product of the two chosen numbers is 600?

A) $1/23$ B) $1/12$ C) $1/2$ D) $12/23$ E) NOTA

For questions 5-7, consider two dice that are to be rolled.

5. The product of the rolled faces of the dice is to be noted. What is the expected value of this product?

A) 7 B) 12.25 C) 15 D) 17.25 E) NOTA

6. The non-negative difference of the rolled faces of the dice is to be noted. What is the expected value of this difference?

A) $35/18$ B) $7/3$ C) $49/18$ D) $28/9$ E) NOTA

7. The greater of the two rolled faces of the dice is to be noted (in the event of doubles, just note the number common to both faces). What is the expected value of this number?

A) $151/36$ B) $77/18$ C) $157/36$ D) $161/36$ E) NOTA

8. A family of mathematicians has *n* children, some (but not all) of whom are boys. They find the probability of a family having the same number of boys and girls that they do when considering all different birth orders of the sexes of the children (in other words, the sequence). The family decides to adopt one more boy, and they found that when they computed a similar probability again with the additional boy, they arrived at the exact same probability (how peculiar!). Originally, how many more girls than boys did the family of mathematicians have?

A) 0 B) 1 C) 2 D) 4 E) NOTA

9. A coin is to be flipped five times. What is the probability than a head is never tossed immediately after a tail?

A) $1/4$ B) $7/32$ C) $3/16$ D) $5/32$ E) NOTA

10. A die is rolled, then a coin is flipped the number of times equivalent to the number rolled on the die. What is the probability that the same number of heads and tails are flipped?

A) $19/96$ B) $9/48$ C) $13/64$ D) $3/16$ E) NOTA

11. One hundred people were surveyed and asked two questions: whether or not they like Transformers and whether or not they like Gobots. Fourty-two people like Transformers only while 12 people like Gobots only. If the two events “likes Transformers” and “likes Gobots” are independent, what is the greater of the two possible numbers of people who like both Transformers and Gobots?

A) 18 B) 24 C) 28 D) 32 E) NOTA

12. Let's skip balls-in-an-urn problems…they are so boring! Instead, try this problem: Vase A contains three blue marbles and two green marbles only, while vase B contains 5 blue marbles and 1 green marble only. A marble is drawn at random from one of the vases and transferred to the other vase, then a marble is drawn from the vase into which the original marble was transferred (each vase is equally likely to be chosen initially). Given that the second drawn marble was blue, what is the probability that the transferred marble was a green marble transferred from vase A to vase B?

A) $185/932$ B) $232/1169$ C) $301/1517$ D) $360/1813$ E) NOTA

13. A real number is chosen from the closed interval $[0,1]$. What is the probability that the distance from the real number to an integer is less than $1/3$?

A) $1/3$ B) $2/3$ C) $1/2$ D) $3/4$ E) NOTA

14. When you think of trash, think of Akeem. At Akeem's university, the University of the United States, 62% of the student body are undergraduate students (as opposed to graduate students), 55% of the student body are male, and 48% of the undergraduate students are male. Determine the probability that a randomly selected student from the University of the United States is male or an undergraduate.

A) 0.69 B) 0.8476 C) 0.8724 D) 0.884 E) NOTA

15. Police plan to enforce speed limits during the morning rush hour on four different routes into the city. The traps on routes A, B, C, and D are in effect 40%, 30%, 30% and 50% of the time, respectively. Speedy Gonzalez always speeds to work, taking one of these routes to work; the probability of Speedy Gonzalez taking routes A, B, C, or D to work on any given day are 0.2, 0.25, 0.1, and 0.45, respectively. What is the probability that Speedy Gonzalez will get a ticket speeding to work?

A) 0.29 B) 0.33 C) 0.37 D) 0.41 E) NOTA

16. For a certain type of light bulb, the probability that the bulb will last at least 1500 hours before burning out is 0.2, and the probability that the bulb will last at least 2000 hours before burning out is 0.15. Find the probability that a light bulb of this type will last at least 2000 hours before burning out, given that it has already lasted 1500 hours without burning out.

A) 0.15 B) 0.35 C) 0.38 D) 0.75 E) NOTA

17. A slight twist on the Monty Hall problem: You are playing a game in which you are to pick one of three doors. Behind one door is A NEW CAR!!!! (Say like Rod Roddy on *The Price is Right*.) Behind the other two doors are goats. The host of this game, Monty Hall, does not know what prize is behind each door. He reveals at random one of the other two doors that was not initially selected. If the car is revealed, the game resets, including the locations of the three prizes behind the doors and you again pick one of the three doors; if a goat is revealed, you have the opportunity to keep your original pick of door or change to the other door whose prize has not yet been revealed, and you win what is behind whichever door you pick at this point. Your goal is to win THE NEW CAR!!!! If you choose to switch doors once a goat is revealed, what is the probability that you will win THE NEW CAR!!!!?

A) $1/3$ B) $1/2$ C) $3/5$ D) $2/3$ E) NOTA

18. Consider the series $\sum_{n=1}^{\infty}\left((2n+3)\left(a/b\right)^n\right)$, where a and b are integers such that $1 \le a < b \le 5$ and the fraction a/b is reduced. Find the probability that such a fraction makes the sum of the series an integer.

A) $7/10$ B) $2/3$ C) $3/5$ D) $7/9$ E) NOTA

19. A dartboard consists of a regular octagon inscribed in a square such that every other side of the octagon lies on a side of the square. If Gary "The Flying Scotsman" Anderson throws three darts at this dartboard, and all three darts hit the board, and all three throws are independent, what is the probability that exactly two of the three darts land inside the octagon?

A) $136-96\sqrt{2}$ B) $168-118\sqrt{2}$ C) $204-144\sqrt{2}$ D) $252-177\sqrt{2}$ E) NOTA

20. The six black face cards are removed from a deck of cards and dealt into a sequence. They are then shuffled and re-dealt into a new sequence. What is the probability that none of the cards in the new sequence are in the same position as they were in the original sequence?

A) $53/144$ B) $91/144$ C) $49/144$ D) $95/144$ E) NOTA

21. A regular n-gon has an exterior angle whose measure is an integer number of degrees. Find the number of possible values of n.

A) 18 B) 20 C) 22 D) 24 E) NOTA

22. OK, I had to have a balls-in-an-urn problem. An urn initially contains 4 red balls, 4 black balls, and some number of other-colored balls. One ball is drawn at a time under the following conditions: if a red ball is drawn, replace it in the urn and add a black ball to the urn; if a black ball is drawn, do not replace it in the urn, but add a red ball to the urn; and if an other-colored ball is drawn, do not replace it in the urn, with no other changes. If the probability that the first three balls drawn are a red, a black, and an other-colored ball, in any order, is $\frac{3068}{13005}$, how many other-colored balls were originally in the urn?

A) 2 B) 4 C) 6 D) 8 E) NOTA

23. Moe, Larry, and Curly take turns rolling a die, in this order. Moe wins if he rolls a prime number, Larry wins if he rolls a composite number, and Curly wins if he rolls neither a prime nor a composite number. If Moe is the first to roll, how many more times likely is Moe to win than Curly?

A) 9 B) 7 C) 5 D) 3 E) NOTA

24. The popular board game Spot It!$_{TM}$ consists of 55 cards, each of which contains 8 distinct symbols. If any two of the cards are selected, they have in common one and only one symbol (for example, there are not two cards that both have a tree AND a star on them). What is the minimum number of symbols needed for the board game Spot It!$_{TM}$?

A) 55 B) 57 C) 59 D) 61 E) NOTA

25. Romeo wants to meet Juliet at the local poison shop so that they will be on the same page when it comes to this whole staged/real death scenario. The shop is open from 8 am until 5 pm. Romeo is always too hasty in his actions, though, and he shows up at some point no later than 1 pm, staying for exactly one hour from the time he shows up so as not to arouse suspicion. Juliet drags her feet when it comes to revival, and she shows up at some point no earlier than 12 noon, staying for at least two hours. If both Romeo and Juliet are at the shop at the same time for any length of time, their lives will no longer end in tragedy; what is the probability that this occurs? Neither Romeo nor Juliet can arrive at the shop before it opens, and neither can stay at the shop after it closes?

A) $1/5$ B) $4/25$ C) $1/10$ D) $2/25$ E) NOTA

26. Bartholomew Sympsonne was a money minter back in the 12th century for King Henry I of England. Unfortunately for the king, Bartholomew was also a shyster. When he minted coins, Bartholomew would put 50 coins in each of 50 boxes; however, one of the 50 coins in each box was a forgery. Knowing Bartholomew was untrustworthy (why the king left Bartholomew in this position, one will never know), the king decided to pull one coin at random from each box to check it for authenticity. Bartholomew felt that there was no way his ruse would be detected (that is, at least one of the coins pulled from one of the boxes was the forgery, thus alerting the king). The probability that the ruse would go undetected is closest to which of the following?

A) $1/e$ B) $1/(e-1)$ C) $2/e^2$ D) $2/(e^2-1)$ E) NOTA

27. Mrs. Brady has three children, one of whom is a daughter born in the fall and one of whom is a daughter born in the summer. What is the probability that Mrs. Brady has three daughters? Assume each season and each sex is equally likely to be the birth season or sex, respectively, of the other child. Also, assume Mrs. Brady did not have any multiple-births.

A) $1/3$ B) $3/8$ C) $1/2$ D) $3/7$ E) NOTA

28. My friends and I have a variant card hand when playing certain 7-card stud poker games. It's called the "Uncle Jesse", and it consists of three pairs with a seventh card not of the same rank as any card in any of the pairs. When determining which hands beat the Uncle Jesse and which hands the Uncle Jesse beats, the probability of being dealt an Uncle Jesse in seven cards is determined, then the calculated probability determines the hand's placement in the rank of hands. For example, if the probability of being dealt an Uncle Jesse was higher than a 5-card straight but lower than a 5-card flush, then the Uncle Jesse would beat a straight but lose to a flush. Additionally, the Uncle Jesse is the only hand that uses more than five cards and is the only additional hand added to the standard poker hands. Further, all 5-card hands' probabilities are calculated based on being dealt seven cards. What 5-card hand is the least in the order of hands that beats the Uncle Jesse?

A) flush B) full house C) four-of-a-kind D) straight flush E) NOTA

29. A die is rolled four times, and the sequence of numbers rolled is noted. What is the probability that the sum of the four outcomes of the roll of the die is exactly 14?

A) $73/648$ B) $83/648$ C) $113/648$ D) $143/648$ E) NOTA

30. On game night at the apartment, Leonard and Sheldon each have a pair of six-sided dice that they will throw in order to determine who gets to set the temperature on the thermostat (contrary to the roommate agreement). Leonard's dice are standard dice, but Sheldon's dice are not standard (such is the essence of Sheldon). Sheldon's dice have a single positive integer on each of their faces, and, curiously enough, the probability distribution of the sum of the faces rolled for Sheldon's dice are exactly the same as that of Leonard's dice. Find the largest possible product of faces rolled for Sheldon's dice.

A) 30 B) 32 C) 35 D) 36 E) NOTA