Where applicable, "E) NOTA" indicates that none of the above answers is correct.

1. Let $X = \{x \in \mathbb{Z}^+ \mid x+1 \text{ is a multiple of } 3\}$ and $Y = \{y \in \mathbb{Z}^+ \mid y \le 50 \text{ and } y+1 \text{ is a multiple of } 4\}$. Find $X \cap Y$.

A) $\{11,23,35,47\}$ B) $\{z \in \mathbb{Z}^+ \mid z+1 \text{ is a multiple of } 12\}$ C) $\mathbb{Z}^+$ D) $\varnothing$ E) NOTA

2. How many elements does the set $\{x \in \mathbb{Z}^+ \mid x^2 - 33 \text{ is a perfect square}\}$ contain?

A) 0 B) 1 C) 2 D) 3 E) NOTA

3. Consider the following two statements:

I) $\mathbb{Z} \cup \mathbb{Q} = \mathbb{Q}$
II) $\mathbb{Z} \cap \mathbb{Q} = \mathbb{Z}$

Either one of these two statements directly imply which of the following statements?

A) $\mathbb{Z} \setminus \mathbb{Q} = \mathbb{Z}$ B) $\mathbb{Q} \subseteq \mathbb{Z}$ C) $\mathbb{Q} \setminus \mathbb{Z} = \mathbb{Q}$ D) $\mathbb{Z} \subseteq \mathbb{Q}$ E) NOTA

4. Suppose $A = \{0, 1, \text{blue}, \text{hot dog}\}$. Which of the following is not an element of 2^A?

A) hot dog B) $\{\text{hot dog}\}$ C) $\{1, \text{blue}\}$ D) $\varnothing$ E) NOTA

5. How many of the following statements are true?

I) Every ordinal is a cardinal.
II) Every cardinal is an ordinal.

A) 0 B) 1 C) 2 D) Dr. Seuss E) NOTA

6. How many proper subsets does $X = \{4,13,17,9\}$ have?

A) 14 B) 15 C) 16 D) 17 E) NOTA

7. Which is the correct sequence of signs that fill in the blanks of the following statement?

$$|\mathbb{Z}^+| \_\_\_\_ |\mathbb{Z}| \_\_\_\_ |\mathbb{Q}| \_\_\_\_ |\mathbb{Z} \times \mathbb{Z}|$$

A) <,=,= B) <,<,< C) =,<,= D) =,=,= E) NOTA

8. Is $Z = \{x \mid x \text{ is a set}\}$ a set in Zermelo-Fraenkel Set Theory with Axiom of Choice (ZFC)?

A) yes B) no C) yes and no D) my head hurts E) NOTA

9. In ZFC, if a non-empty set *X* is a subset of its power set, what must be true?

A) $X \subseteq \varnothing$ B) $2^X = X$ C) *X* must contain infinitely many elements
D) this is impossible—only $\varnothing$ is a subset of its power set E) NOTA

10. Consider $T = \{x \mid x \text{ is a jelly doughnut}\}$. Which of the following is not correct about *T*?

A) $T \notin T$ B) $T \in T$ C) $T \subseteq T$ D) $T \neq \varnothing$ E) NOTA

And now, five questions about infinite ordinals and cardinals:

11. When the ordinals are ordered in the normal way, ω_0 ("omega-0") is the least infinite ordinal. In other words, there is no ordinal immediately preceding ω_0 – if there was, it would have to be finite, meaning there would be a greatest finite ordinal, implying a greatest integer, an obvious impossibility. Because of this, ω_0 is generally classified as what type of ordinal?

A) topological B) maximum C) successor D) limit E) NOTA

12. Find the correct sequence of ordinal in the first blank and cardinal in the second blank for the following statement:

$$\omega_0 + 1 = \_\_\_\_\_, \text{ while } |\omega_0 + 1| = \_\_\_\_\_$$

A) ω_0, ω_0 B) $\omega_0, \omega_0 + 1$ C) $\omega_0 + 1, \omega_0$ D) $\omega_0 + 1, \omega_0 + 1$ E) NOTA

13. Find the correct sequence of ordinal in the first blank and cardinal in the second blank for the following statement:

$$1 + \omega_0 = \_\_\_\_\_, \text{ while } |1 + \omega_0| = \_\_\_\_\_$$

A) ω_0, ω_0 B) $\omega_0, 1+\omega_0$ C) ω_0+1, ω_0 D) $1+\omega_0, 1+\omega_0$ E) NOTA

14. Suppose that κ and τ are distinct infinite cardinals. Assuming ZFC, which of the following must equal $\kappa+\tau$?

A) 2^{ω_0} B) ω_0 C) the greater of κ and τ D) 2^{α}, where α is the greater of κ and τ
E) NOTA

15. Assuming ZFC, the continuum hypothesis suggests no set exists whose cardinality lies strictly between ω_0 and 2^{ω_0}—in other words, $2^{\omega_0}=\omega_1$. Due to this, ω_1 is the least cardinal (and ordinal) that is ______. Find the word that correctly fits this sentence.

A) infinite B) uncountable C) countable D) unnatural E) NOTA

16. The Cantor Set is a subset of the real interval $[0,1]$ found in the following way:

1) Remove the middle third of the interval $[0,1]$, keeping what remains.
2) Remove the middle thirds of the remaining intervals, keeping what remains.
3) Repeat step 2 ad infinitum; the Cantor Set is what remains after all removals.

Which of the following numbers is not a member of the Cantor Set?

A) $1/4$ B) $3/10$ C) $10/13$ D) $27/40$ E) NOTA

17. If $A\cup B=A\cap B$, which of the following must be true?

A) $A\cap B=\varnothing$ B) $B\subset A$ C) $A\subset B$ D) $A=B$ E) NOTA

18. A group is a set G with an operation $*$ on G such that for all $a,b\in G$, $a*b\in G$ that also has three other properties. Which of the following is not one of those properties?

A) G contains an identity element e such that for all $a\in G$, $a*e=e*a=a$
B) For each $a\in G$, there exists $a^{-1}\in G$ such that $a*a^{-1}=a^{-1}*a=e$ (the identity element)
C) $a*(b*c)=(a*b)*c$ for all $a,b,c\in G$ D) $a*b=b*a$ for all $a,b\in G$ E) NOTA

19. Avoiding theory too much, there is a sporadic group called the “Monster group” or the “Friendly Giant”, so-called because it has a very large cardinality—what is the cardinality?

A) 7920 B) 37,357,657,400 C) 130,661,359,751,492,312,718,400,000
D) 808,017,424,794,512,875,886,459,904,961,710,757,005,754,368,000,000,000
E) NOTA

For questions 20-22, consider the sets $A=\{2,3,5,7,8\}$ **,** $B=\{2,3,4,6,9\}$ **, and** $C=\{3,4,5,7,9\}$ **.**

20. Find $A \,\Delta\, B$.

A) $\{x\in\mathbb{Z} \mid 2\le x\le 9\}$ B) 0 C) $\{x\in\mathbb{Z} \mid 4\le x\le 9\}$ D) -1 E) NOTA

21. Find $\left(A \,\Delta\, (B\setminus C)\right) \Delta \left(A \,\Delta\, (B \,\Delta\, C)\right)$.

A) $\{5,7\}$ B) 4 C) $\{4\}$ D) $\{3,6,8\}$ E) NOTA

22. Find $\left(\bigcup_{n\in\{A,B,C\}} n\right) \Delta \left(\bigcap_{n\in\{A,B,C\}} n\right)$.

A) $\{x\in\mathbb{Z} \mid 2\le x\le 9\}$ B) $\{3,6,8\}$ C) $\{x\in\mathbb{Z} \mid 4\le x\le 9\}\cup\{2\}$ D) 7 E) NOTA

23. The finite ordinals, which are labeled exactly the same as the nonnegative integers, are defined as follows:

1) $0=\varnothing$
2) $n=\{0,1,\ldots,n-1\}$ for all positive integers n

Which of the following statements is not true?

A) $n\in n+1$ for any nonnegative integer n B) $n \,\Delta\, (n+1)=n$ for any nonnegative integer n

C) $n\subseteq n+1$ for any nonnegative integer n D) $n+1=n\cup\{n\}$ for any nonnegative integer n

E) NOTA

24. If an element is in the complement of $A\cup B$, then that element must be in both the complement of *A* and the complement of *B*. Additionally, if an element is in the complement of $A\cap B$, then that element must be in at least one of the complement of *A* and the complement of *B*. These two laws of set theory are known collectively as whose laws?

A) De Morgan B) Boole C) Peirce D) Murphy E) NOTA

25. Paraphrasing a Yogi Berra quotation, "Baseball is 90% mental and the other half is physical." If this is true, and if baseball is composed solely of mental and physical aspects, what percentage of baseball is both mental and physical?

A) 10% B) 30% C) 40% D) 50% E) NOTA

26. Each person in a club of 66 people plays at least one of the following classic Nintendo games: Excitebike, Metroid, Burgertime, and Castlevania. The number of people who play which games are given in the following table:

Game(s) played	Number of students
Excitebike	30
Metroid	34
Burgertime	31
Castlevania	26
Excitebike, Metroid, & Burgertime	5
Excitebike, Metroid, & Castlevania	3
Excitebike, Burgertime, & Castlevania	5
Metroid, Burgertime, & Castlevania	5
Excitebike, Metroid, Burgertime, & Castlevania	1

How many people play exactly two of the four games?

A) 7 B) 11 C) 19 D) 24 E) NOTA

For questions 27 and 28, consider $X=\{a,b,c\}$, all three elements being distinct.

27. Given a set *S*, a topology *T* on that set is a collection of subsets of *S* such that:

1) $\varnothing \in T$ and $S \in T$
2) the union of any number of elements of *T* is itself an element of *T*
3) the intersection of a finite number of elements of *T* is itself an element of *T*

Which of the following is not a topology on *X*?

A) $T=\{\varnothing,\{a,b\},\{c\},X\}$ B) $T=\{\varnothing,\{c\},\{a\},X\}$ C) $T=\{\varnothing,\{c\},\{a,c\},\{b,c\},X\}$
D) $T=\{\varnothing,\{a\},\{c\},\{a,c\},\{b,c\},X\}$ E) NOTA

28. Among all topologies on *X*, let *M* be the maximum number of elements of a single topology on *X*, and let *m* be the minimum number of elements of a single topology on *X*. Find the value of $M-m$.

A) 3 B) 4 C) 6 D) 8 E) NOTA

29. Quaternions are a number system that extends the complex numbers. The basis elements of the quaternions are 1, *i*, *j*, and *k*, where *i*, *j*, and *k* satisfy the following equations:

$$i^2 = j^2 = k^2 = ijk = -1$$

One notable feature of the quaternion number system is that multiplication is not commutative. Given the above equations, find the quaternion product *ji*.

A) $-k$ B) k C) -1 D) 1 E) NOTA

30. Because of the basis elements described in the previous equation, all quaternions are of the form $a+bi+cj+dk$, where a, b, c, and d are real numbers. Two quaternions may be multiplied together by distributing (in the correct order, of course, since there is no commutativity) and determining the product of the basis elements (as you hopefully did for *ji* in the previous problem)—this product is called a Hamilton product. Find the Hamilton product for $(1+2i-j+k)(-2+i+j-3k)$.

A) $i-2j-6k$ B) $7i+2j+2k$ C) $i+2j+6k$ D) $7i-2j-2k$ E) NOTA