Answers

1. 9
2. 6
3. $\frac{1}{18}$
4. 4,064,255
5. 19
6. 6
7. 226,801
8. 18
9. $\frac{9\pi}{2}$
10. 144
11. 505
12. 1008
13. 110
14. 10π
15. 72^{o}
16. 126
17. 220 and 284
18. -6
19. 2
20. $y=-\frac{1}{2}x+4$
21. $-6<x<1$
22. 4
23. 720
24. $\frac{7}{18000}$
25. 2304π

Solutions:

1. $(3-6)^2 = (-3)^2 = 9$

2. 2^{2016} is a multiple of 4, so $2^{2^{2016}}$ is also a multiple of 4, so $2^{2^{2^{2016}}}$ will end in a 6.

3.
$$x = Log_{a^6} \sqrt[3]{a}$$
$$\left(a^6\right)^x = a^{\frac{1}{3}}$$
$$a^{6x} = a^{\frac{1}{3}}$$
$$6x = \frac{1}{3}$$
$$x = \frac{1}{18}$$

4. $\frac{(n+1)!}{(n-2)!n} = \frac{(n+1)(n)(n-1)(n-2)!}{(n-2)!n}$, so when $n = 2016$, the quotient is $= (n+1)(n-1) = n^2 - 1$

$2016^2 - 1 = 4,064,255$

5.
$$-3x^5 + 3x^4 - 9x^3 + x^2 + 2x + 5$$
$$x = -1$$
$$-3(-1)^5 + 3(-1)^4 - 9(-1)^3 + (-1)^2 + 2(-1) + 5$$
$$= -3(-1) + 3(1) - 9(-1) + 1 + (-2) + 5$$
$$= 3 + 3 + 9 + 1 - 2 + 5$$
$$= 19$$

6. $$\begin{vmatrix} 1 & 0 & 2 & -1 \\ -1 & 0 & 0 & 0 \\ 2 & 2 & 0 & 2 \\ -2 & 1 & -1 & 0 \end{vmatrix} = -(-1)\begin{vmatrix} 0 & 2 & -1 \\ 2 & 0 & 2 \\ 1 & -1 & 0 \end{vmatrix} = 0 + 4 + 2 - 0 - 0 - 0 = 6$$

7. $1+2+3+\ldots+673 = \frac{673\times 674}{2} = 226,801$

8. 14, 24, 34, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 54, 64, 74, 84, 95

So 18.

9.
$$r = 1yd = 3ft$$
$$h = 6in = \frac{1}{2}ft$$
$$V = \pi r^2 h$$
$$= \pi(3)^2(\frac{1}{2}) = \frac{9\pi}{2}$$

10.
$$f(2) + f(-2) = (2^3 - 2)(3(2^2))(2+5) + \left((-2)^3 - 2\right)\left(3(-2)^2\right)(-2+5)$$
$$= (6)(12)(7) + (-10)(12)(3) = 504 + (-360) = 144$$

11. The line has equation $y + 8 = \frac{4+8}{2+1}(x+1) \Rightarrow y = 4x - 4$, so

$4a - 4 = 2016 \Rightarrow 4a = 2020 \Rightarrow a = 505$.

12.
$$(1+i)^4 = ((1+i)^2)^2 = (1+2i+i^2)^2 = (1+2i-1)^2 = (2i)^2 = -4$$
$$\Rightarrow (1+i)^8 = 16 = 2^4 \Rightarrow (1+i)^{2016} = \left((1+i)^8\right)^{252} = \left(2^4\right)^{252} = 2^{1008}$$
$$\log_2 2^{1008} = 1008$$

13. $\frac{2+0.8n}{10+n} = \frac{3}{4} \Rightarrow 8 + 3.2n = 30 + 3n \Rightarrow 0.2n = 22 \Rightarrow n = 110$

14. Find the side of the square
$$A = 50 = S^2$$
$$S = 5\sqrt{2}$$

Then the diagonal of the square which is the diameter of the circle. $D = S\sqrt{2}$, $D = \left(5\sqrt{2}\right)\sqrt{2}$, $D = 10$ so

perimeter is $P = \pi D = 10\pi$

15. $3:3:9$
$3x + 3x + 9x = 180$
$15x = 180$
$x = 12$
$9x = 108^{\circ}$

The supplement of 108 is **72**.

16. 6, 7, 9, 12, 16, 21, 27, 34, 42, 51, 61, 72, 84, 97, 111, **126**

17. A well-known result in number theory, the smallest amicable pair is 220 and 284.

18. $\sqrt{2x^2 - 7} = 3 - x$
$2x^2 - 7 = 9 - 6x + x^2$
$x^2 + 6x - 16 = 0$
$(x+8)(x-2) = 0$
$x = -8$
or
$x = 2$
, so the sum of the solutions is -6

19. To have integral enclosed area, the two equal side lengths must be odd while the remaining side length must be even. Call the odd length 2k+1, then the other length is either 1) 2k+2 or 2) 2k. The altitude to the even base has length 1) $\sqrt{k(3k+2)}$ or 2) $\sqrt{(k+1)(3k+1)}$, and for the perimeter to be less than 100, 1) k<16 or 2) k<17 (for integral lengths. A quick check yields one solution for each category: 1) k=2 or 2) k=8. Therefore, there are two solutions.

20.
$$y = mx + b$$
$$y = \left(-\frac{1}{2}\right)x + b$$
$$3 = \left(-\frac{1}{2}\right)(2) + b$$
$$3 = -1 + b$$
$$4 = b$$
$$y = \left(-\frac{1}{2}\right)x + 4$$

21.
$$10x + 9 < 7x + 12$$
$$3x < 3$$
$$x < 1$$
$$7x + 12 < 8x + 18$$
$$-6 < x$$

These two inequalities don't exclude any solutions from each other, so the solution is -6<x<1.

22.
$$c + s = 24$$
$$3c + 2.5s = 62$$
$$3c + 2.5s - (2.5c + 2.5s) = 62 - (2.5(24))$$
$$.5c = 62 - 60 = 2$$
$$c = 4$$

23. $\frac{{}_nP_r}{{}_nC_r} = \frac{\frac{n!}{(n-r)!}}{\frac{n!}{r!(n-r)!}} = r!$, so $\frac{{}_{2016}P_6}{{}_{2016}C_6} = 6! = 720$

24.
$$0 = 12(1-5x)^2 - 17|1-5x| + 6 = (3|1-5x| - 2)(4|1-5x| - 3)$$
$$1 - 5x = \pm\frac{2}{3} \text{ or } \pm\frac{3}{4}$$
$$x = \frac{1}{15},\ \frac{1}{3},\ \frac{1}{20},\ \text{or } \frac{7}{20} \Rightarrow \text{ product} = \frac{7}{18{,}000}$$

$1 - 5x = 11$		$1 - 5x = -11$
$-10 = 5x$	or	$12 = 5x$
$-2 = x$		$12/5 = x$

$$V = \frac{4}{3}\pi r^3$$

25. $$= \frac{4}{3}\pi\left(12\right)^3$$

$$= \frac{4(1728)\pi}{3} = 2304\pi$$