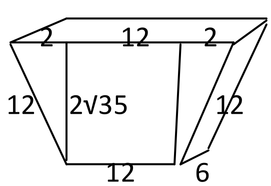
Theta Geo 3-D Solns

1. 96/6 = 16 area of one face; therefore each side is 4. V = 4^3 = 64 ans: C
2. Length of space diagonal is what is needed. D = $\sqrt{3^2+4^2+6^2}$ = $\sqrt{61}$ ans: D
3. $100\pi = 4\pi r^2$; r = 5. $V = 4\pi 5^3/3 = 500\pi/3$ ans: B
4. The original cone had a height of $6\sqrt{3}$, radius 6, and slant height 12. Its lateral surface area $\pi rl/2 = 72\pi$. The cone that was cut off had a height of $6\sqrt{3}-8$, radius $6-8/\sqrt{3}$, and slant height 12– $16/\sqrt{3}$; its lateral surface area $(344-192\sqrt{3})\pi/3$. The difference between these lateral areas is $72\pi - (344-192\sqrt{3})\pi/3 = {(-128+192\sqrt{3})\pi}/{3}$ ans: B

5. $48\pi = 2\pi r$; radius of the cross section is 24. The 7 and 24 are legs of a right triangle whose hypotenuse is the radius of the sphere meaning the radius of the sphere is 25. Surface Area of the sphere is $4\pi r^2 = 2500\pi$ ans: A
6. The skewer is the altitude from the right angle to the hypotenuse (the diameter of the disk). It is the geometric mean between the segments of the hypotenuse, one of which is 1. The other segment would have to be 12 making the diameter of the disk 13. Area = $\pi\left(\frac{13}{2}\right)^2 = 169\pi/4$ Ans: E
7. To calculate the length of the walking stick you must find the radius to the center of the torus and add the 3 cm radius of the metal of the torus. First calculate the arc length of the center of the handle. If straightened out the handle would be 15 cm long since half the volume of the wooden part of the cane. That means the desired arc length(semicircle) is 15 and the radius of that semicircle is $15/\pi$ add the 3 cm of the metal above this arc makes the handle have a height of $15/\pi + 3$. Adding the length of the wood makes the total height of the cane $15/\pi + 33$. Ans: D
8. Volume of the cube is 64 the portion removed has volume (.5)(4) = 2. Ice 64-2 = 62 Ans: A
9. Right triangle area is $.5l^2$=.5; l^2=1; each leg is 1 meaning the hypotenuse is $\sqrt{2}$. The lid has dimensions 4 x $\sqrt{2}$ and area $4\sqrt{2}$ Ans: C

10. $A_{trapezoids}=2(28\sqrt{35}) = 56\sqrt{35}$; $A_{rectangles}$=3(72)=216; A_{straps}=2(29)=58;

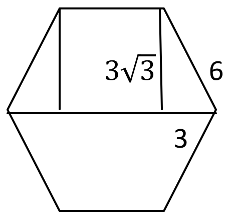
    $A_{binding}$=perimeters of top and bottom+legs=44+36+4(12)=128 $T_{area}=402+56\sqrt{35}$

    Ans: D

11. All that is needed is to figure the vertical height of the slanted blade (4in =1/3 ft) plug it and the radius (2ft) into the volume of a cylinder formula. $V = \pi r^2 h = \pi(2)^2(1/3) = 4\pi/3$ Ans: D
12. The width of the noisemaker would be 5 cm to finish the right triangle. $V = \pi r^2 h = \pi(12)^2(5) = 720\pi$ Ans: B

13. The solid formed is made of two congruent cone frustums joined together by their larger bases. The formula for the volume of a frustum of a cone is $V = {}^{h}/_{3}\left(B + b + \sqrt{Bb}\right)$.



B has a radius of 6; b has a radius of 3; h = $3\sqrt{3}$

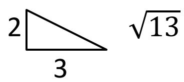
$V = {}^{3\sqrt{3}}/_{3}\left(36\pi + 9\pi + \sqrt{45\pi^2}\right) = 63\pi\sqrt{3}$ x 2 = $126\pi\sqrt{3}$

Ans: C

14. D = $\sqrt{l^2 + w^2 + h^2} = \sqrt{8^2 + 10^2 + 12^2} = \sqrt{308} = \sqrt{4(77)} = 2\sqrt{77}$ Ans: D

15. $D = a\sqrt{3} = 10; a = 10/\sqrt{3}$; Open topped cube has 5 squares side $10/\sqrt{3}$; A =5a^2= 500/3

Ans: D

16. 

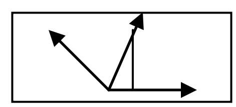
SA = B + .5pl = $4\pi + 2\pi\sqrt{13}$

Ans: C

17. Arc length = $\frac{\theta}{360}(2\pi r) = \frac{40}{360}(2\pi 8) = \frac{16\pi}{9}$

Ans: A

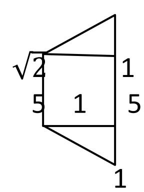
18. 10 is the vertical segment below and is the long leg of 30-60-90 triangle. The length of the hypotenuse $\left(20/\sqrt{3}\right)$ is the answer. Ans: E



19. 1 ft = .25 $(2\pi r)$; r = $2/\pi$;

$V = {}^{4}/_{3}(\pi)\left(\frac{2}{\pi}\right)^3 = \frac{32}{3\pi^2}$ Ans: C

20. This solid is composed of two cones and one cylinder. All of them have radius of 1. Surface area of the solid equals The lateral areas of the two cones and the lateral area of the Cylinder. $A = 2\left(\pi(1)\sqrt{2}\right) + 2\pi(1)(5) = 2\pi\sqrt{2} + 10\ \pi$



Ans: C

21.

	179°	59′	60″
-	15°	31′	4″
	164°	28′	56″

Ans: A

22. $6\pi = \frac{\theta}{360}(2\pi 9)$; $6\pi = \frac{\pi\theta}{20}$; $\theta = 120°$ That makes the isosceles triangle with legs 9 and vertex angle 120. Drawing the altitude to the base forms two 30-60-90 right triangles whose hypotenuse is 9 and long leg is $4.5\sqrt{3}$ The length of segment AB is the sum of the two long legs: $9\sqrt{3}$. Ans: C

23. First find the slant height of a face. The altitude of the face forms two right triangles whose hypotenuse is 13 and leg on the base is 5, that leads to other leg (the slant height) of 12. Now use the right triangle formed by the apothem (5) of the base, the altitude of the pyramid, as legs and the slant height (12)of a face as the hypotenuse. Using the Pythagorean theorem gives a height of $\sqrt{119}$. Ans: D

24. The brace is effectively the chord of a 100 degree arc of a circle. Draw a radius to each end of the brace to form an isosceles triangle; draw the altitude, which is also the angle bisector and median to the base, to form congruent right triangles with a 50° opposite the side formed by half the brace. The leg opposite the 50° angle has length 2 and the hypotenuse is the length we want to find. $sin50 = {}^{2}/_{h}$; $h = {}^{2}/_{sin50}$ Ans: E

25. Two of the walls are congruent trapezoids with bases of 10 and 8 and height of 15. Each of their areas is 135 for a total of 270. 630-270 = 360 left of the rectangles. Letting the unknown length be "x" gives the equation 8x +10x = 360; and x the distance between the two trapezoidal walls is 20. The room is a prism with trapezoids for bases and the distance between the bases is 20. $V = Bh$; $V = 135(20) = 2700$ Ans: B

26. Let half the base edge be "b" which will also be the apothem of the square base. Drawing the slant height and the altitude of the pyramid along with drawing the apothem will form a right triangle with short leg "b", long leg (the altitude) $b\sqrt{3}$, and hypotenuse (the slant height) "2b". The slant height(2b), half a base edge(b), are legs of the right triangle whose hypotenuse is the lateral edge (200). $(2b)^2 + b^2 = 40000$; $b^2 = 8000$; b = $40\sqrt{5}$. That means that the altitude ($b\sqrt{3}$) is $40\sqrt{15}$ and the height of the first floor is $20\sqrt{15}$. Ans: D

27. The exterior angle of 135 means that the triangle is 45-45-90. The hypotenuse is 1 ft. and each leg is ${}^{\sqrt{2}}/_{2}\ ft$. One leg is the diameter and the other is all but 2 inches (${}^{1}/_{6}\ ft$) of the height.

 Radius = ${}^{\sqrt{2}}/_{4}$ and height = ${}^{(3\sqrt{2}+1)}/_{6}$ $V = Bh = \pi\left(\frac{\sqrt{2}}{4}\right)^2\left(\frac{3\sqrt{2}+1}{6}\right) = \pi\left(\frac{1}{8}\right)\left(\frac{3\sqrt{2}+1}{6}\right)$ Ans: B

28. The altitude of the cone is 15 (long leg of 30-60-90 triangle). $V_{cone} = \frac{(25\pi)(15)}{3} = 125\pi$. Each side of the cube tank is $10\sqrt{2}$. The volume displaced by the cone would be $(10\sqrt{2})^2h = 125\pi$; $200h = 125\pi$; h = ${}^{5\pi}/_{8}$ Ans: C

29. The figure formed is a pyramid with an isosceles right triangle as a base and two congruent 30-60 right triangles that share AB as their short leg. BC and BD are each $10\sqrt{3}$ and are the legs of the isosceles right triangle. Triangle ABC = ABD = $50\sqrt{3}$ Triangle BCD =150 Triangle ACD is an isosceles triangle with legs 20 and base CD that is the hypotenuse of BCD and = $10\sqrt{6}$. That makes the area of ACD = $50\sqrt{15}$ Surface area = $50\sqrt{3} + 50\sqrt{3} + 50\sqrt{15} + 150 = 100\sqrt{3} + 50\sqrt{15} + 150$ Ans: E

30. Connecting the centers of the three spheres forms an equilateral triangle whose area can be found by $\frac{a^2\sqrt{3}}{4} = 4\sqrt{3}$. $a^2 = 16$; $a = 4$ That means the radius of a sphere is 2.

 $A = 4\pi r^2 = 16\pi$ Ans: C