For all questions, E "NOTA" means none of the above answers is correct.

1. If *b* buildings can be constructed in *c* minutes, how many minutes does it take to construct *d* buildings? Assume all buildings are constructed at a constant rate.

a)
$$\frac{dc}{b}$$
 b) $\frac{bc}{d}$ c) $\frac{c}{d}$ d) $\frac{bd}{c}$ e) NOTA

- 2. In the early 1600s, Johannes Kepler proposed that the path of a planet orbiting a sun is elliptical, with the sun at one of the foci. Consider a planet in elliptical orbit around a sun with the sun located at focus F_1 . While in orbit, the closest the planet comes to F_1 is 140 million kilometers. The farthest the planet goes from F_1 is 260 million kilometers. The eccentricity of this planet's orbit is 0.28. What is the length of the minor axis, in millions of kilometers, of the ellipse that represents the planet's orbit?
 - a) 192 b) 384 c) $16\sqrt{481}$ d) $8\sqrt{2451}$ e) NOTA
- 3. Consider a circle-shaped paper that is folded twice– first into a semicircle, then a quartercircle. Then, one flap of the quartercircle is opened up to form a cone shape. What is the angle θ , in degrees, of the bottom of the cone when we look at it from the side?



4. A gambler played the following game with a friend. The gambler bet half the money in his pocket on the toss of a coin; he won on heads and lost on tails. The game was repeated, each time for half the money held by the gambler (i.e. if the gambler wins, he has $\frac{3}{2}$ of his previous amount. If he loses, he has $\frac{1}{2}$). At the end, the number of times the gambler lost was equal to the number of times he won, which was not zero. The gambler:

a) Broke evenb) Lost moneyc) Gained moneyd) Not Enough Informatione) NOTA

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- 5. In counting *x* colored M&M's, some blue and some green, it was found that 45 of the first 50 counted were blue. After this, the remaining M&M's were counted in groups of 10, and 7 M&M's in each of these groups were blue. Find the maximum possible value of *x* if 80% or more of the M&M's counted were blue.
- a) 90 b) 100 c) 110 d) 120 e) NOTA
- 6. *Yersinia pestis* is a highly infectious bacterium that was responsible for the Bubonic plague. A microbiologist is modeling new culture growth each minute with the summation below. Find this sum.

$$\sum_{n=1}^{\infty} \left(10 \cdot \frac{2n}{2^n} \right)$$

a) 35 b) 45 c) 50 d) 60 e) NOTA

- 7. Uranium-235 has a half-life of 710 million years. If it is determined that a certain amount of stored U-235 will be considered safe only when its radioactivity has dropped to 0.10 percent of the original level $\left(\frac{A}{A_o} = \frac{1}{1000}\right)$, which of the following is closest to the amount of time in years the U-235 must be stored securely to be safe?
- a) 8.5×10^8 b) 3.2×10^9 c) 7.1×10^9 d) 8.5×10^9 e) NOTA
- 8. Two teachers, one male and one female, are taking their 8 students, 4 males, 4 females, out to lunch on a field trip. They are all seated around a circular table that has 10 seats. How many ways can they be seated if both teachers sit together and all the males and females alternate?
- a) 72 b) 576 c) 1152 d) 9600 e) NOTA
- 9. What is the probability that a randomly chosen distinguishable permutation of the letters in the word MOSQUITOS has U and I together?
- a) $\frac{1}{8!}$ b) $\frac{1}{9}$ c) $\frac{2}{9}$ d) $\frac{1}{3}$ e) NOTA

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- 10. In Chemistry and Biology, it is common to find the pH of a solution using the following formula: $pH = -\log_{10}[H^+]$, where $[H^+]$ represents the concentration of hydrogen ions. With this knowledge, which of the following is closest to the hydrogen ion concentration in a solution at pH 5.7? Hint: $\log_{10} 2 \approx .3$
- a) $1 x 10^{-6}$ b) $2 x 10^{-6}$ c) $1 x 10^{-5}$ d) $7 x 10^{-5}$ e) NOTA
- 11. Newton's law of universal gravitation states that any two bodies in the universe attract each other with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. Two bodies are at a distance of D. If the force exerted is quartered and the masses remain the same, what is the new distance in terms of D?
- a) $\frac{D}{4}$ b) $\frac{D}{2}$ c) 2D d) 4D e) NOTA
- 12. Three perfectly spherical balls are tightly packed into a cylindrical can, where the balls are stacked on top of each other. What fraction of the can is not taken up by the balls?
- a) $\frac{2}{9}$ b) $\frac{1}{4}$ c) $\frac{1}{2}$ d) $\frac{2}{3}$ e) NOTA
- 13. A man looks at an analog clock and notices that the time is exactly 3:15 PM. How many additional minutes must he wait from that moment until the minute hand and the hour hand form a 90 degree angle?
- a) 15 b) $\frac{195}{11}$ c) $\frac{360}{11}$ d) $\frac{420}{11}$ e) NOTA
- 14. A speaker talked for sixty minutes to a full classroom. Twenty percent of the audience heard the entire talk and ten percent slept through the entire talk. Half of the remaining seventy percent heard one third of the talk while the other half heard two thirds of the talk. What was the average number of minutes of the talk heard by members of the audience?
- a) 30 b) 33 c) 34 d) 36 e) NOTA

- 15. A chemist has x liters of an acidic solution that is x% acid. How many liters of pure acid must be added to make a solution that is 2x% acid?
- a) $\frac{x^2}{1-2x}$ b) $\frac{2x}{100-2x}$ c) $\frac{x^2}{100-2x}$ d) $\frac{x^2}{100+2x}$ e) NOTA
- 16. Each child in a family has at least five brothers and at least six sisters. What is the smallest number of children the family could have?
- a) 10 b) 11 c) 12 d) 13 e) NOTA
- 17. A telephone number has the form *ABC*. *DEF*. *GHIJ*, where each letter represents a different digit. The digits in each part of the number are in decreasing order; that is, A > B > C, D > E > F, and G > H > I > J. Furthermore, *D*, *E*, and *F* are consecutive even digits, *G*, *H*, *I*, and *J* are consecutive odd digits, and A + B + C = 9. Find *A*.
- a) 4 b) 5 c) 6 d) 8 e) NOTA
- 18. At a fair, a game of chance is played. Three fair coins are tossed simultaneously. If all three land on heads, the player wins \$15. If all three land on tails, the player wins \$10. Otherwise, no money is won. The game costs \$5 to play. What is the expected amount of net <u>profit</u> for a player that plays the game twice?
- a) \$-.625 b) \$-1.25 c) \$-3.75 d) \$6.25 e) NOTA
- 19. At the end of 1994 Walter was half as old as his grandmother. The sum of the years in which they were born is 3838. How old will Walter be at the end of 1999?
- a) 48 b) 50 c) 53 d) 55 e) NOTA

20.Some marbles in a bag are red and the rest are blue. If one red marble is removed, then oneseventh of the remaining marbles are red. If two blue marbles are removed instead of one red, then one-fifth of the remaining marbles are red. How many marbles were in the bag originally?

a) 18 b) 22 c) 36 d) 57 e) NOTA

- 21. The mean of three numbers is exactly ten more than the least of the numbers and exactly fifteen less than the greatest of the three. If the median of the three numbers is 5, then the sum of the three numbers is
 - a) 5 b) 20 c) 25 d) 30 e) 36
- 22. A book has pages numbered starting at 1 using consecutive positive integers. The digit 3 is printed 237 times during page number printing. What is the greatest number of pages the book could have?
 - a) 663 b) 664 c) 672 d) 673 e) 683
- 23. Sneha is taking eggs to the market to sell. The eggs are in a cart that holds up to 500 eggs. If the eggs are removed from the cart either 2, 3, 4, 5, or 6 at a time, one egg is always left over. If the eggs are removed 7 at a time, no eggs are left over. Let n denote the number of eggs in the cart. Which of the following is true about n?
 - a) $n \in [1, 100]$ b) $n \in [101, 200]$ c) $n \in [201, 300]$ d) $n \in [301, 400]$ e) NOTA
- 24. Tony notices that a sequence of three real numbers forms an arithmetic progression with a first term of 9. He also notices that if 2 is added to the second term and 20 is added to the third term, the three resulting numbers form a geometric progression. What is the smallest possible value for the third term of the geometric progression?
 - a) 1 b) 4 c) 36 d) 49 e) NOTA
- 25. A round sign is divided into seven sections as in the diagram. The sign is to be painted with five colors (blue, green, orange, red and yellow) so that each section is painted with one color and no adjacent sections are the same color. Also, each color must be used at least once and no color can be used more than twice. In how many different ways can the sign be painted? Assume no two of the seven sections are congruent, and therefore are distinguishable.



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- 26. Jalen and Nick live 13 miles apart. Yesterday, Jalen started to ride his bike toward Nick's house. A little later, Nick started to ride his bike toward Jalen's house. When they met, Jalen had ridden for twice the length of time as Nick and at four-fifths of Nick's rate. How many miles had Nick ridden when they met?
 - a) 4 b) 5 c) 6 d) 7 e) NOTA
- 27. An envelope contains 8 bills in dollar amounts: 2 ones, 2 fives, 2 tens and 2 twenties. Krishna draws two bills at random without replacement. What is the probability that the sum of the bills he draws is at least \$20?
 - a) $\frac{1}{4}$ b) $\frac{2}{7}$ c) $\frac{3}{7}$ d) $\frac{2}{3}$ e) NOTA
- 28. Three cowboys entered a cafe. The first ordered 4 sandwiches, a cup of coffee, and 10 doughnuts for \$8.45. The second ordered 3 sandwiches, a cup of coffee, and 7 doughnuts for \$6.30. How much did the third cowboy pay for a sandwich, a cup of coffee, and a doughnut (in dollars)?
 - a) \$2.00 b) \$2.05 c) \$2.10 d) \$2.15 e) NOTA
- 29. A farmer has 200 yards of fencing material. What is the largest rectangular area he can enclose (in square yards) if he wants to use a 4 yard wide gate that does not need to be covered by the fencing material (it will be built of a different type of material)?
 - a) 2,500 b) 2,601 c) 2,704 d) 2,809 e) NOTA

30. On a fence are sparrows and pigeons. When five sparrows leave, there remains two pigeons for every sparrow. After that, twenty-five pigeons leave, and the ratio of sparrows to pigeons becomes three to one. Find the original number of birds.

a) 44 b) 48 c) 50 d) 54 e) NOTA