



12. Points  $H(-1,0)$ ,  $J(3,-5)$ , and  $K(1,4)$  are the midpoints of the sides of  $\triangle XYZ$ . If the coordinates of  $X$ ,  $Y$ , and  $Z$  are  $(a,b)$ ,  $(c,d)$ , and  $(e,f)$ , then what is the value of  $a + b + c + d + e + f$ ?
- A. 2                      B. 4                      C. 6                      D. 8
13. Quadrilateral  $ABCD$  has coordinates  $A(0,0)$ ,  $B(6,1)$ ,  $C(6,3)$ , and  $D(-2,5)$ . Find the coordinates of a point  $P$  that minimizes the sum  $AP + BP + CP + DP$ .
- A.  $(\frac{5}{2}, \frac{9}{4})$               B.  $(\frac{1}{2}, \frac{5}{2})$               C.  $(4,2)$               D.  $(2,4)$
14. Cali has begun to form a triangle by choosing vertices at  $(5,-1)$  and  $(-3,5)$ . She asks Valentina for a third ordered pair to complete her triangle. As it turns out, Valentina suggests a point  $(a,b)$  which doesn't complete a triangle at all. What is the value of  $3a + 4b$ ?
- A. 11                      B.  $3\sqrt{41}$                       C.  $\sqrt{269}$                       D. 17
15. After the epic fail Valentina had in the previous problem, she recovers by selecting a different point  $(x,y)$  to complete the triangle (using her new point and Cali's two points), and chooses a point which actually makes the triangle a right triangle. Which is the most accurate description of the locus of points in the Cartesian plane from which Valentina could have chosen her new point?
- A. A Line                      B. Two parallel lines  
C. Two perpendicular lines                      D. A circle and two parallel lines
16. A regular octagon  $ABCDEFGH$  has vertices  $A(1,0)$  and  $B(5,0)$ . If the coordinates of vertices  $E$  and  $F$  are  $(h,i)$  and  $(j,k)$  respectively, then find the value of the product  $hijk$ .
- A. 640                      B. 320                      C.  $240 + 160\sqrt{2}$                       D.  $160 + 160\sqrt{2}$
17. Carlos begins a journey at the origin on the coordinate plane. He heads east one unit, north 2 units, west 3 units, south four units, and progresses each time traveling exactly 1 more unit than the previous segment after turning  $90^\circ$  counterclockwise. What are Carlos's coordinates at the moment he has travelled a total of 2016 units?
- A.  $(-32,-31)$               B.  $(-32,-30)$               C.  $(-31,32)$               D.  $(-32,32)$
18.  $\triangle XYZ$  consists of vertices  $X(0,15)$ ,  $Y(0,0)$ , and point  $Z$  on the positive  $x$ -axis. Let  $n$  be the value of the greatest possible  $x$ -coordinate for the point  $Z$  such that the perimeter of  $\triangle XYZ$  does not exceed 60. What is the sum of the digits of the number  $n$ ?
- A. 1                      B. 2                      C. 3                      D. 10
19. The circles  $(x-3)^2 + (y+2)^2 = 90$  and  $(x+5)^2 + (y-3)^2 = 91$  intersect at points  $A$  and  $B$ . Find the slope of  $\overline{AB}$ .
- A.  $-2$                       B.  $-\frac{3}{4}$                       C.  $\frac{4}{5}$                       D.  $\frac{8}{5}$
20. How many lattice points (points with integer coordinates) are in the region  $x^2 + y^2 \leq 19$ ?
- A. 56                      B. 57                      C. 60                      D. 61

21. Rocco is building a linear fence on his ranch. The grid location of the first post he installs is at  $(-3, -8)$ , and the fifth post he installs is found at  $(9, 12)$ . If he continues in the same direction, always maintaining equal spacing between posts, then what are the coordinates of the 63<sup>rd</sup> post Rocco installs?
- A.  $(180, 297)$       B.  $(183, 302)$       C.  $(186, 307)$       D.  $(189, 312)$
22. A concrete plant is located at the point  $(18, 2)$ . A job site is located at  $(6, 10)$ . After leaving the concrete plant, the concrete truck must stop somewhere along the x-axis to get sand and somewhere along the y-axis to get water before going to the job site. The driver wants to minimize the total distance traveled, so he stops at  $(a, b)$  to get sand, and stops at  $(c, d)$  to get water. What is the value of  $a + b + c + d$ ?
- A. 13      B. 15      C. 17      D. 21
23. Yorly plays a game where she begins at point  $A_1$  with coordinates  $(2015, 2016)$ . She then progresses from point  $A_n(x_n, y_n)$  to the point  $A_{n+1}(x_{n+1}, y_{n+1})$  using the following rules for coordinates:
- a.) if  $x_n$  is even, then  $x_{n+1} = \frac{x_n}{2}$ ,  
b.) if  $x_n$  is odd, then  $x_{n+1} = 2x_n$ ,  
c.) if  $y_n$  is even, then  $y_{n+1} = \frac{y_n}{2}$ ,  
d.) if  $y_n$  is odd, then  $y_{n+1} = 2y_n$ .
- What are Yorly's coordinates when she is standing at point  $A_{2016}$ ?
- A.  $(4030, 63)$       B.  $(4030, 126)$       C.  $(2015, 63)$       D.  $(2015, 126)$
24. Three consecutive vertices of a convex kite ABCD are  $A(-1, 5)$ ,  $B(2, 4)$ , and  $C(3, 1)$ . What are the coordinates of the fourth vertex, given that the area enclosed by the kite is 32 square units?
- A.  $(-4, -2)$       B.  $(-7, -5)$       C.  $(-5, -3)$       D.  $(-6, -4)$
25. A region is bounded by the lines  $y = 0$ ,  $x = -1$ ,  $x = 2$ , and  $2x + 3y = 13$ , and this region is revolved about the line  $x = 2$ . What is the volume of the resulting solid?
- A.  $52\pi$       B.  $45\pi$       C.  $39\pi$       D.  $33\pi$
26. If the same region described in problem 25 was revolved instead about the line  $y = 0$ , then what is the volume of the resulting solid?
- A.  $\frac{117\pi}{4}$       B.  $49\pi$       C.  $52\pi$       D.  $147\pi$
27. Jonathan and Charlie are known to ride their bikes at an equal speed of  $8\sqrt{5}$  miles per hour. Jonathan leaves the point  $(-1, 6)$  at the same moment Charlie leaves the point  $(3, -2)$ , and they are both headed for the point  $(-3, -5)$ . To the nearest minute, how many minutes will Charlie have to wait on Jonathan before he arrives? Assume a unit scale on either axis is equivalent to one mile.
- A. 8      B. 15      C. 18      D. 23
28. Find the degree measure of the acute angle between the lines  $x - 2y = 5$  and  $x + 3y = 5$ .
- A.  $22\frac{1}{2}$       B. 30      C.  $52\frac{1}{2}$       D. 45

29. Given two circles of radius length 4 centered at  $(3,0)$  and  $(19,0)$ , there is one tangent line drawn that is tangent to both of these two circles which also intersects the positive x-axis and the positive y-axis. If the equation of this tangent line is written in the form  $Ax + 3y = C$  where A and C are real numbers, what is the value of the product  $3AC$ ?

A. 99

B. 33

C.  $99\sqrt{3}$

D.  $33\sqrt{3}$

30. Consider  $A(4,4)$ ,  $B(0,8)$  and the line  $3x - 4y = 0$ . If point  $C(x, y)$  is on line segment  $\overline{AB}$  such that the distance from C to the given line is exactly 3, then find the value of  $\frac{y}{x}$ .

A.  $\frac{59}{47}$

B.  $\frac{47}{59}$

C.  $\frac{17}{39}$

D.  $\frac{39}{17}$