Answers:

1. C
2. B
3. C
4. B
5. B
6. A
7. B
8. C
9. A
10. B
11. E (0)
12. A
13. C
14. A
15. D
16. C
17. D
18. B
19. D
20. D
21. B
22. D
23. A
24. D
25. C
26. B
27. B
28. D
29. A
30. D

Solutions:

1. $\sqrt{(7--5)^2+(-5-4)^2+(6--2)^2} = \sqrt{12^2+(-9)^2+8^2} = \sqrt{289} = 17$

2. We want a line perpendicular to line *b*. We know the slope of a line in standard form is $-\frac{A}{B}$, but we want the perpendicular slope, which is the opposite reciprocal slope, namely $\frac{B}{A}$. This means that we just switch A and B, and change one of their signs. This can be done easily to the original equation, giving $x+2y=C$, and we plug in the point $(2,3)$ to arrive at $C=8$, and the equation of the line is $x+2y=8$.

3. This problem is easy to think about using "vector" thinking. In order to move from $(2,3)$ to $(8,12)$, one must travel 6 units right and 9 units up, which is represented by the vector $\langle 6,9\rangle$. Now P divides the segment into a 2:3 ratio, this means AP:AB=2:5, so to get to point P, we want to go $\frac{2}{5}$ of the way from A to B, and $\frac{2}{5}$ of the vector $\langle 6,9\rangle$ is $\langle\frac{12}{5},\frac{18}{5}\rangle$. This means we move from point A to the right $\frac{12}{5}$ and up $\frac{18}{5}$. The point P is then $(2+\frac{12}{5},3+\frac{18}{5})$, or $(\frac{22}{5},\frac{33}{5})$, and $\frac{x}{y}=\frac{2}{3}$.

4. We know this line is tangent to the circle at (3,4), which means the line passes through the point (3,4) and is perpendicular to the radius at the point of tangency. The slope of the radius, which contains the center (0,0) and the point (3,4) is $\frac{4}{3}$. This means we want a line perpendicular, which will have slope $-\frac{3}{4}$, and pass through (3,4). This line is $3x+4y=25$, which has x-intercept $\left(\frac{25}{3},0\right)$ and y-intercept $\left(0,\frac{24}{4}\right)$. The area of the triangle is then $A=\frac{1}{2}bh=\frac{1}{2}\left(\frac{25}{3}\right)\left(\frac{25}{4}\right)=\frac{625}{24}$.

5. Using the midpoint formula gives A the coordinates $\left(\frac{1}{2},4\right)$ and $\frac{1}{2}+4=\frac{9}{2}$.

6. Using the distance formula: $\sqrt{(-5-x)^2+(x-2)^2}=13$. Square both sides to get $(-5-x)^2+(x-2)^2=169$ then simplify and divide by 2 to get $x^2+3x-70=0$, which has solutions $x=-10\ or\ 7$, which have a product of -70. (or use Vieta's formulas for the final step)

7. The median will have one endpoint at B(-6,9), and the other endpoint will be the midpoint of $\overline{AC}$, which has coordinates (-3,0). Use the distance formula then to get BE = $\sqrt{90}=3\sqrt{10}$.

8. The centroid is found by averaging the x-coordinates, then averaging the y-coordinates. D then has coordinates $\left(\frac{2-6-8}{3}, \frac{8+9-8}{3}\right)$ or $(-4,3)$. We then find the distance from (-4,3) to (-8,-8) using the distance formula, which will give us $\sqrt{137}$.

9. In order to cut these two regions into two congruent pieces each, we must pass through each of their "centers." (Note: this is not possible to do with every shape, but works nicely for rectangles and circles.) The center of the rectangle is $\left(\frac{-3+7}{2}, \frac{13+5}{2}\right)$ or $(2,9)$. The circle has standard form equation $(x-12)^2+(y+5)^2=16$ and thus has center (12, -5). The slope between (2,9) and (12, -5) is $-\frac{7}{5}$.

10. Finding the slopes of the sides, we get a slope of 2 for both $\overline{AB}$ and $\overline{CD}$, and we get slopes of $-\frac{1}{2}$ for both $\overline{AD}$ and $\overline{CB}$. This means that opposite sides are parallel, meaning it certainly belongs to the parallelogram family. However, we also can see that adjacent sides are perpendicular, so it is a rectangle. We then rule out the possibility of the figure being a square by seeing that adjacent sides are not congruent, or by noting that the diagonals are not perpendicular. Rectangle is the most precise name.

11. The given point is actually on the given line, so the distance is 0.

12. There are (at least) two ways to do this. **Method 1:** Draw the coordinates on the coordinate plane. Since segment HI is a midsegment, then it is parallel to one of the sides of the triangle, and it is half of the length of that side. Thus, I travel the same slope and distance in both directions from point J (which is the midpoint of the side parallel to segment HI) to arrive at points (5, -1) and (1, -9). Then I use the fact that segment IJ is a midsegment, and travel "up" and "down" that slope and distance from point H, which gives the new point (-3, 9) and confirms one of the points already listed. Thus, ΔXYZ has coordinates (5, -1), (1, -9), and (-3, 9), and the sum of those coordinates is 2. **Method 2:** The midpoints are given by $\left(\frac{a+c}{2}, \frac{b+d}{2}\right), \left(\frac{a+e}{2}, \frac{b+f}{2}\right), and \ \left(\frac{e+c}{2}, \frac{f+d}{2}\right)$. When you add these up you discover that $a+b+c+d+e+f$ is also equal to the sum of all of coordinates of the midpoints as well, 2.

13. If we think about AP + CP, we know that if P is between A and C, then AP + PC = AC. We also know from the triangle inequality that if P is not between A and C, then AP + PC > AC. We are trying to minimize the sum, so we know P must be between A and C. By similar reasoning, we determine that P must be between B and D. So in order to minimize the sum of those 4 segments, point P must be the intersection of the diagonals of the quadrilateral, which is (4,2).

14. In order to fail to complete a triangle, the point Valentina chose must be on the line containing the first two points. This line is 3x + 4y = 11, so for any point on that line, specifically the one chosen in the form (a, b), then 3a + 4b = 11.

15. If the two points Cali chose are to be used as endpoints of a leg of the triangle, then the third point can be anywhere on the two lines passing through one of the original two points and perpendicular to the line containing the original two points. This gives us two parallel lines. If the original two points are endpoints of the hypotenuse, then the third point must be on the circle having the two original points as endpoints of a diameter. We also know that we must exclude the two original points, because Valentina would fail again if she chose one of those. Therefore, the most accurate description is two parallel lines and a circle (with only two points excepted from that locus).

16. We know all the sides have length 4, and we know moving from point B to point C, we will travel 4 units at an angle of 45° with the positive x-axis. This means that we will us a 45°-45°-90° triangle to see we will travel $2\sqrt{2}$ units to the right and $2\sqrt{2}$ units up. To get from C to D, we simply travel 4 units up. Then from D to E, we travel $2\sqrt{2}$ units up and $2\sqrt{2}$ units to the left, which puts us directly above point B again and at the point $\left(5, 4+4\sqrt{2}\right)$. Then from E to F, we just move left 4 units to arrive at $\left(1, 4+4\sqrt{2}\right)$. So $(1)\left(4+4\sqrt{2}\right)(5)\left(4+4\sqrt{2}\right) = (5)\left(48+32\sqrt{2}\right) = 240+160\sqrt{2}$.

17. After the first 4 moves, Carlos is at (-2, -2), and after 8 moves, he is at (-4, -4), so after 4n moves, he is at (-2n, -2n). We must figure out how many segments he must move in order to travel 2016 units. We want to know the greatest integer *x* such that, 1+2+3+…+x≤2016. It turns out that 2016 is actually the 63rd triangular number, so x=63, and he will move 63 entire segments – lucky for us to end exactly on a triangular number. We also know 60 is a multiple of 4, namely 4(15), so we end at the point (-30,-30) after his first 60 segments. We then move 61 units east to point (31,-30), then move 62 units north to point (31,32), then 63 units west to point (-32, 32).

18. Let the side along the x-axis have length *n*. Then the hypotenuse has length $\sqrt{n^2+15^2}$. Then we know the perimeter is less than 60 so we use the following series of equations: $15+n+\sqrt{n^2+15^2} \leq 60 \Rightarrow \sqrt{n^2+15^2} \leq 45-n \Rightarrow n^2+15^2 \leq (45-n)^2$ (since both sides of the inequality are positive) $\Rightarrow n^2+225 \leq n^2-90n+2025 \Rightarrow 90n \leq 1800 \Rightarrow n \leq 20$. So the greatest value for *n* is 20, and 2+0 = 2.

19. The centers of the two circles are (3,-2) and (-5,3) and the slope between the centers is $-\frac{5}{8}$. Since the points A and B lie on the same circle, then the center of the circle (or of both circles)

must lie on the perpendicular bisector of $\overline{AB}$. Then the slope of $\overline{AB}$ is perpendicular to the slope of the segment between the centers, and $\overline{AB}$ has a slope of $\frac{8}{5}$.

20. A drawing is helpful here, but if we consider only the points in Quadrant I, as well as the positive x-axis, we can then use symmetry to finish the problem. We know that for any point inside the circle $x^2 + y^2 < 19$, and if y=0, then x could be 1, 2, 3, or 4. If y=1, then x could also be 1, 2, 3, or 4. If y=2, then x could be 1, 2, or 3. If y=3, then x could be 1, 2, or 3, and if y=4, x could only be 1. This gives us a total of 4+4+3+3+1=15 points in Quadrant I plus the positive x-axis. So giving all 4 Quadrants plus the 4 total axis pieces gives 60 points. The origin makes 1 more, so there are a total of 61.

21. Rocco begins at (-3, -8) and the fifth post is at (9, 12). So 4 moves takes him right 12 units and up 20 units. This means each move will take him right 3 and up 5. To get to the 63rd post he must make 62 moves, which will take him right 186 and up 310 from (-3, -8) in order to arrive at (183, 302).

22. Just think 1 dimension at a time. From the concrete plant, he must travel down 2 units to get to the x-axis and up 10 units to get to (6,10) for a total of 12 units in the y-direction. In the x-direction, he must travel 18 units to the y-axis, then back 6 units to get to (6,10) for a total of 24 units. This means his total trip requires traveling 24 units horizontally while traveling 12 units vertically. This means we will travel 2 units horizontally for every 1 unit vertically as we go, which means we hit the x-axis at (14, 0) and then hit the y-axis at (0, 7), and 14+0+0+7=21. (You can also do the problem by reflecting (6,10) across both axes and drawing a straight path to (18,2). This will make the x-intercept visible. By reflecting (18,2) across both axes and drawing a path to (6,10), the y-intercept will be evident.

23. You can follow the rules for the coordinates individually, or you can do both at the same time. Focusing on the x-coordinate gives $x_1 = 2015, x_2 = 4030, x_3 = 2015, and\ x_4 = 4030$. The alternating pattern gives $x_{2016} = 4030$. The y-coordinate takes slightly more persistence as $y_1 = 2016, y_2 = 1008, y_3 = 504, y_4 = 252, y_5 = 126, y_6 = 63, y_7 = 126, y_8 = 63$. Once again we reach an alternating pattern, so $y_{2016} = 63$. The final point is (4030, 63).

24. We know the area of a kite can be found as $\frac{1}{2}d_1d_2$. Using the distance formula, we know AC = $4\sqrt{2}$. So $A = \frac{1}{2}d_1d_2 = \frac{1}{2}(4\sqrt{2})d_2 = 32 \Longrightarrow d_2 = 8\sqrt{2}$. Now since $\overline{AC}$ has slope -1, and the diagonals of a kite are perpendicular, then the slope of $\overline{BD}$ is 1, and we must travel $8\sqrt{2}$ units, which specifically has us moving down 8 units and left 8 units from point B to (-6, -4).

25. The line 2x+3y=13 intersects the lines x = -1 and x = 2 at the points (-1, 5) and (2, 3), respectively. The other two points bounding our region are (2, 0) and (-1, 0). When we rotate about x = 2, then we end up with a cylinder with a cone cut out of it. The cylinder has height 5 and radius 3, with $V = \pi r^2 h = \pi 3^2(5) = 45\pi$. The cone has radius 3 and height 2, so $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi 3^2(2) = 6\pi$. So the volume of the revolved solid is $45\pi - 6\pi = 39\pi$.

26. We keep the same points for the bounded region above, but revolve around the x-axis, which creates a frustum with height 3 and bases with radii 5 and 3. For a frustum, we know $V = \frac{1}{3}h\left(B_1 + B_2 + \sqrt{B_1 B_2}\right) = \frac{1}{3}(3)\left(25\pi + 9\pi + \sqrt{(25\pi)(9\pi)}\right) = \frac{1}{3}(3)(25\pi + 9\pi + 15\pi) = 49\pi$.

27. Using the distance formula, we know Jonathan must travel $5\sqrt{5}$ units, and Charlie must travel $3\sqrt{5}$ units. Since they move at the same rate, when Charlie arrives, Jonathan will still have $2\sqrt{5}$. Since he travels $8\sqrt{5}$ in an hour, it will take him $\frac{2\sqrt{5}}{8\sqrt{5}} = \frac{1}{4}$ of an hour, or 15 minutes.

28. We can use the slopes to find the tangent of the angle between the lines. The first line has slope $\frac{1}{2}$ while the second line has slope $-\frac{1}{3}$. Then, use the formula $\tan\theta = \frac{m_1 - m_2}{1+m_1 m_2} = \frac{\frac{1}{2}-\left(-\frac{1}{3}\right)}{1+\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1$. So then angle must be 45°.

29. There are four tangent lines to these two circles, but the only one which crosses the positive x- and the positive y-axis is the common internal tangent with negative slope. Since the circles are the same size, the common internal tangent passes through the midpoint between the two centers of the circles, which means we know the line passes through (11, 0). Also, by drawing the circle centered at (3,0), and by drawing the radius to the point of tangency of this internal tangent line, we know the radius is 4, and the distance between (3, 0) and (11, 0) is 8 units.

This gives us a 30°-60°-90° triangle with the tangent line, the radius, and the positive x-axis. We also know the 30° angle is at the vertex (11, 0), which means the 60° angle is at (3, 0). So we must go 4 units at 60° to get to the point of tangency. Drawing an altitude to the hypotenuse will help us see what we should do. This means we will go 2 units right (the short leg of the smaller triangle), and we will go $2\sqrt{3}$ units up from (3, 0) to arrive at $(5, 2\sqrt{3})$.

Now we know the line passes through (11, 0) and $(5, 2\sqrt{3})$. The slope formula will give us $m = -\frac{\sqrt{3}}{3}$ and the equation of the line matching the format given is $\sqrt{3}x + 3y = 11\sqrt{3}$. This means $3AC = 3(\sqrt{3})(11\sqrt{3}) = 99$.

30. We can first write the equation of $\overleftrightarrow{AB}$ as $x + y = 8$. Then using the fact that $y = -x + 8$, as well as the formula for the distance from a point to a line, we get the distance from the point $(x, -x + 8)$ to the line $3x - 4y + 0 = 0$ is:

$$\frac{|Ax + By + C|}{\sqrt{A^2 + B^2}} = \frac{|3x - 4y + 0|}{\sqrt{3^2 + 4^2}} = \frac{|3x - 4(-x + 8) + 0|}{\sqrt{3^2 + 4^2}} = \frac{|3x + 4x - 32|}{5} = \frac{|7x - 32|}{5} = 3$$

Then $|7x - 32| = 15$ and $7x - 32 = 15$ or $7x - 32 = -15$. This gives us possible x-values of $\frac{47}{7}$ or $\frac{17}{7}$. Only one of these is between 4 and 0, so only one of them can be on $\overline{AB}$, which is $\frac{17}{7}$. Plugging back into $x + y = 8$, we get $y = \frac{39}{7}$, which means $\frac{y}{x} = \frac{39}{17}$.