

For all questions, answer choice "E. NOTA" means none of the above answers is correct.

1. Two sides of a triangle are $\sqrt{2}$ and $\sqrt{3}$. The inclusive angle between the two sides is 30° . Find the area enclosed by the triangle.

A. $\frac{\sqrt{6}}{4}$ B. $\frac{\sqrt{6}}{2}$ C. $\frac{\sqrt{6}}{3}$ D. $\frac{\sqrt{6}}{6}$ E. NOTA

2. A survey showed the following results:

84 people like the music of Lady Gaga only

34 people like the music of Ludacris only

72 people like the music of Lil' Wayne only

A total of 100 people like only two of the three artists.

85 disliked all three.

The number of people surveyed is the largest perfect square less than 1000. How many people like the music of all three artists?

A. 876 B. 686 C. 586 D. 296 E. NOTA

3. If 2137^{753} is multiplied out, what would the units' digit be?

A. 1 B. 7 C. 9 D. 3 E. NOTA

4. What is the diameter of a circle which has the same enclosed area as $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$?

A. ab B. $(ab)^2$ C. \sqrt{ab} D. $2\sqrt{ab}$ E. NOTA

5. $\odot P$ and $\odot R$ have radii of $9\sqrt{2}$ and $\sqrt{2}$ respectively. The length of the common internal tangent between the two circles is $\sqrt{22}$. Find the distance between the centers of $\odot P$ and $\odot R$.

A. $\sqrt{222}$ B. $5\sqrt{6}$ C. $25\sqrt{3}$ D. $\sqrt{151}$ E. NOTA

6. What is the domain of the following function?

$$f(x) = \frac{x + 6}{x - 7}$$

A. $\{x \in \mathcal{R} \mid x > -6 \text{ or } x < -6\}$ B. $\{x \in \mathcal{R} \mid x > -7 \text{ or } x < -7\}$
 C. $\{x \in \mathcal{R} \mid x > 7 \text{ or } x < 7\}$ D. $\{x \in \mathcal{R} \mid x > 6 \text{ or } x < 6\}$ E. NOTA

7. The arithmetic mean of two numbers is 4.5. The geometric mean of the same two numbers is $2\sqrt{2}$. Find the sum of the squares of these same two numbers.
- A. 81 B. 9 C. $1 + 2\sqrt{2}$ D. 65 E. NOTA
8. How many diagonals exist in a regular icosagon?
- A. 113 B. 120 C. 170 D. 180 E. NOTA
9. In a nine-element data set of integers from 0 to 100 inclusive, the mode is 14, the median is 38, and the range is 77. What is the largest possible value of the mean?
- A. $\frac{499}{9}$ B. 54 C. 55 D. $\frac{497}{9}$ E. NOTA
10. What is the sum of the last two digits of $\sum_{n=1}^{2016} n^2$?
- A. 15 B. 10 C. 13 D. 6 E. NOTA
11. Evaluate the expression: $\begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -6 & -2 & 7 \\ 3 & 1 & 1 \\ -9 & 4 & 5 \end{bmatrix}$
- A. $\begin{bmatrix} -96 & -82 & 77 \\ 63 & 51 & 41 \\ -39 & 24 & 15 \end{bmatrix}$ B. $\begin{bmatrix} 15 & 10 & 0 \\ 3 & 4 & 3 \\ 12 & -2 & -4 \end{bmatrix}$
- C. $\begin{bmatrix} -93 & 18 & 106 \\ -57 & 9 & 67 \\ -21 & 0 & 28 \end{bmatrix}$ D. $\begin{bmatrix} 3 & 6 & 14 \\ 9 & 6 & 5 \\ -6 & 6 & 6 \end{bmatrix}$ E. NOTA
12. Given that $a = \log 2$ and $b = \log 3$, expand $\log x$, where x is the number of distinct five-letter permutations of the letters in the term WASHU.
- A. $a + 2b + 1$ B. $2a + 2b + 1$
 C. $2a + b + 1$ D. $a + b$ E. NOTA
13. How many intersection points over the interval $[-2\pi, 2\pi]$ do the graphs of $y = \cos(2x)$ and $y = \frac{1}{2}$ have?
- A. 2 B. 4 C. 8 D. 16 E. NOTA

14. Determine the explicit formula for the n^{th} term of the arithmetic sequence -18, -8, 2, ...

- A. $a_n = 28 - 10n$ B. $a_n = a_{n-1} + 10n$
 C. $a_n = a_{n-1} - 10n$ D. $a_n = -28 + 10n$ E. NOTA

15. Determine the maximum possible real root for the following equation over the interval $\frac{\pi}{6} \leq \theta \leq \frac{\pi}{2}$:

$$0 = x^2 - 2 \sin(\theta) x + \frac{\sin^2(2\theta)}{4}$$

- A. 0 B. 2 C. $\frac{3+2\sqrt{3}}{4}$ D. $\frac{2+2\sqrt{3}}{3}$ E. NOTA

16. Solve for x : $\frac{\log(0.0625)}{\log(0.1)} = \frac{\log(4^x)}{\log(9)}$

- A. 2 B. -2 C. 1 D. -1 E. NOTA

17. What is the sum of the elements in the thirteenth row of Pascal's Triangle? The first row consists of a single 1.

- A. 2048 B. 4096 C. 1024 D. 8192 E. NOTA

18. Express the base-5 number $(4102)_5$ in base-8.

- A. 1117_8 B. 1114_8 C. 1014_8 D. 1017_8 E. NOTA

19. What is the maximum number of points of intersection for a set of five distinct circles?

- A. 10 B. 15 C. 20 D. 25 E. NOTA

20. Find $c + d$ if $\frac{1}{\sqrt[3]{2}-\sqrt[3]{3}} = \frac{\sqrt[3]{a}+\sqrt[3]{b}+\sqrt[3]{c}}{d}$ for $c > b > a > 0$, and c and d are relatively prime integers.

- A. 8 B. 9 C. -1 D. 1 E. NOTA

21. If $i = \sqrt{-1}$, what is the reciprocal of the conjugate of the complex number $3i^{14} - 5i^{23}$?

- A. $\frac{1}{3-5i}$ B. $\frac{1}{-3+5i}$ C. $\frac{1}{-3-5i}$ D. $\frac{1}{3+5i}$ E. NOTA

22. Mr. Hallman drops a bouncy ball from a balcony of height 6 feet. Every time the ball bounces, it bounces to two-thirds of its previous height. How far does the ball travel before it comes to rest?

- A. 6 B. 12 C. 24 D. 36 E. NOTA

23. What is the positive x -intercept of $y = -2 + \ln(x^2 - \frac{1}{9})$?

- A. $\pm(e + \frac{1}{3})$ B. $\sqrt{e^2 + \frac{1}{9}}$ C. $e + \frac{1}{3}$ D. $\sqrt{e + \frac{1}{3}}$ E. NOTA

24. Solve for z : $81^{z-1} = 729$

- A. $5/2$ B. $7/4$ C. 3 D. $1/2$ E. NOTA

25. If $0.223\overline{81} \dots$ is expressed as a reduced fraction of positive integers in the form of $\frac{p}{q}$, then what is $q - p$?

- A. -6731 B. 6731 C. 4269 D. -4269 E. NOTA

26. What are the coordinates of the point on the curve $(x - a)^2 + (y - b)^2 = r^2$ with the minimum x -value?

- A. $(x - r, y)$ B. $(a - r, b)$ C. $(r - a, b)$ D. $(r - x, y)$ E. NOTA

27. At what value(s) of x do the graphs $2x^2 + y^2 = 4$ and $y = 5x^2$ intersect?

- A. $\frac{-1 + \sqrt{101}}{25}$ B. $\frac{1}{5}\sqrt{-1 + \sqrt{404}}$
C. $\pm \frac{1}{5}\sqrt{\frac{-1 + \sqrt{404}}{25}}$ D. $\pm \frac{1}{5}\sqrt{-1 + \sqrt{101}}$ E. NOTA

28. A Swiss Bank ATM pin code consists of five digits, each an integer from 0 to 9, inclusive. In order to be more secure, all five digits must be distinct, but the pin code may not consist of strictly increasing or strictly decreasing numbers when reading the pin code from left to right (so, for example, neither 24689 nor 98642 is an acceptable pin code, but 26489 is acceptable, as is 98462). How many such pin codes are possible?

- A. 29,736 B. 118 C. 14,868 D. 252 E. NOTA

29. It takes 10 hours to fill a cylindrical water tank from a pipe B. Using a different pipe P, it takes 8 hours to fill the same tank. To drain this tank if it was full, it would take 16 hours. Currently, the tank is 40% full. How many more minutes will it take to completely fill the tank if both pipes B and P are left completely open and the drain is left completely open?

- A. $\frac{48}{13}$ B. $28\frac{3}{4}$ C. $76\frac{1}{4}$ D. $221\frac{7}{13}$ E. NOTA

30. Determine the eccentricity of the conic section represented (on a Cartesian grid with standard axes x and y) by the equation $y^2 + 4y - x^2 - 6x - 21 = 0$

- A. -2 B. 2 C. $-\sqrt{2}$ D. $\sqrt{2}$ E. NOTA