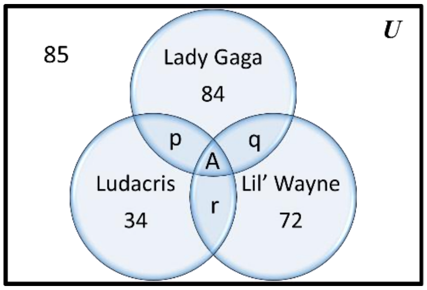
Answers:

1. A
2. C
3. B
4. D
5. A
6. C
7. D
8. C
9. D
10. A
11. D
12. C
13. C
14. D
15. B
16. A
17. B
18. D
19. C
20. A
21. C
22. E
23. B
24. A
25. C
26. B
27. D
28. A
29. D
30. D

Solutions:

1. Use $A_{\Delta ABC}=0.5(ab)\sin c$ to find area. Thus, $A=(0.5)(\sqrt{2})(\sqrt{3})(\sin 30°)=(0.5(\sqrt{6})(0.5)=0.25\sqrt{6}=\mathbf{\frac{\sqrt{6}}{4}}$.Start off by determining your total amount of people. The easiest way is to start at $30^2=900$ and keep going: $31^2=961$, $32^2=1024$… too large, so the number of people is 961.
2. Next, build a Venn diagram to help with visualization:

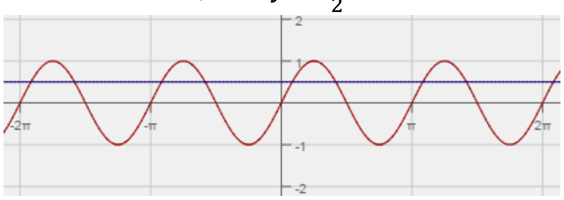
   

   We know that $p+q+r=100$ and $34+84+72+p+q+r+A=964-85$. Seeing that there is $p+q+r$ in both equations, substitute $p+q+r=100$ into the second equation:
   $$34+84+72+(100)+A=961-85$$
   $$290+A=876$$
   $$A=\mathbf{586}$$
3. When multiplied out with *n* as the exponent*,* the units digit goes by the following pattern: *n*=0→1, *n*=1→7, *n*=2→9, *n*=3→3, *n*=4→1,… So a pattern is found to repeat in sets of four. 753 divided by 4 leaves a remainder of 1, so the last digit is **7**.
4. The area of an ellipse is $A=\pi ab$. Setting $\pi ab=\pi r^2$ yields $r=\sqrt{ab}$. So the diameter must be $\mathbf{2\sqrt{ab}}$.
5. The formula for finding the length of the line of centers is $\sqrt{(common\ internal\ tangent)^2+(larger\ radius+smaller\ radius)^2}$. So, plugging in the numbers reveals $\sqrt{(\sqrt{22})^2+(9\sqrt{2}+\sqrt{2})^2}=\sqrt{(\sqrt{22})^2+(10\sqrt{2})^2}=\sqrt{22+100*2}=\sqrt{22+200}=\mathbf{\sqrt{222}.}$
6. The denominator for this function cannot equal zero, or the function would be undefined at that point. Thus setting $x-7=0$, we find that $x=\mathbf{7}$ would be not included in the set.
7. The arithmetic mean is defined as $\frac{a+b}{2}$. The geometric mean is defined as $\sqrt{ab}$. Since we have both of these values, we can use a system of equations to solve for a and b:
   $\begin{cases}\mathbf{\frac{a+b}{2}=4.5}\\ \sqrt{ab}=2\sqrt{2}\end{cases}\rightarrow\begin{cases}\boldsymbol{a+b=9}\\ \sqrt{ab}=2\sqrt{2}\end{cases}\rightarrow\begin{cases}\boldsymbol{a=9-b}\\ \sqrt{ab}=2\sqrt{2}\end{cases}\rightarrow\begin{cases}a=9-b\\ \boldsymbol{(\sqrt{ab})^2}=\boldsymbol{(2\sqrt{2})}^2\end{cases}\rightarrow\begin{cases}a=9-b\\ \boldsymbol{ab=8}\end{cases}\rightarrow$
   $\begin{cases}a=9-b\\ \boldsymbol{(9-b)*b=8}\end{cases}\rightarrow\begin{cases}a=9-b\\ \boldsymbol{9b-b^2=8}\end{cases}\rightarrow\begin{cases}a=9-b\\ \boldsymbol{b^2-9b+8=0}\end{cases}\rightarrow$

$$\begin{cases} a = 9 - b \\ (\boldsymbol{b} - \mathbf{1})(\boldsymbol{b} - \mathbf{8}) = \mathbf{0} \therefore \boldsymbol{b} = \mathbf{1}, \mathbf{8} \end{cases}$$

With solutions for b, find a: when $b = 1, a = 8$; when $b = 8, a = 1$. So it doesn't matter what number is used for which variable; the numbers remain the same to be used for the final computation. The sum of the squares of each number is $1^2 + 8^2 = \mathbf{65.}$

8. A regular icosagon has 20 sides/vertices, referred to in the formula as n. So use the formula for finding the number of diagonals in a regular polygon: $n(n-3)/2$. $\frac{20(20-3)}{2} = 10(17) = \mathbf{170.}$
9. Since the mode is 14, the range is 77, and the median is 38, the following has to be the set of the nine numbers:
$$\{14,14,36,37,38,88,89,90,91\}$$
Adding up all of the numbers equals 497, so the mean is $\frac{\mathbf{497}}{\mathbf{9}}$.
10. $\sum_{n=1}^{2016} n^2 = \frac{(2016)(2017)(4033)}{6}$. The last two digits of the sum are only dependent on the last two digits of the numbers of the product. So, $\frac{16*17*33}{6} = \frac{8976}{6} = 1496$. So $9 + 6 = \mathbf{15.}$
11. Add the elements of each matrix: $\begin{bmatrix} 9+-6 & 8+-2 & 7+7 \\ 6+3 & 5+1 & 4+1 \\ 3+-9 & 2+4 & 1+5 \end{bmatrix} = \begin{bmatrix} \mathbf{3} & \mathbf{6} & \mathbf{14} \\ \mathbf{9} & \mathbf{6} & \mathbf{5} \\ \mathbf{-6} & \mathbf{6} & \mathbf{6} \end{bmatrix}$
12. Permutation calculation: $5! = 120$, so $\log x = 2\log 2 + \log 3 + \log 10 = \mathbf{2\boldsymbol{a} + \boldsymbol{b} + 1}$
13. The period of $y = \cos(2x)$ is π, the absolute value portion flips everything below the x-axis to above it, and $y = \frac{1}{2}$ strikes the curve **8** times, as noted in the figure below:



14. The first term, denoted by $a1$, is –18. Next, find the difference between successive terms:
$$a_2 - a_1 = -8 - (-18) = -8 + 18 = 10$$
$$a_3 - a_2 = 2 - (-8) = 2 + 8 = 10$$
$$a_4 - a_3 = 12 - 2 = 10$$
The common difference, denoted by d, is 10. So, use the format for explicit formulas using the first term:
$$\begin{aligned} a_n &= a_1 + (n-1)d \\ &= -18 + (n-1)10 \\ &= -18 + 10n - 10 \\ &= -28 + 10n \end{aligned}$$
The explicit formula for the nth term is $\boldsymbol{a_n} = \mathbf{-28 + 10\boldsymbol{n}}$.

15. Use the quadratic formula to find the roots of the equation:

$$x = \frac{2\sin(\theta) \pm \sqrt{4\sin^2(\theta) - \sin^2(2\theta)}}{2}$$
$$x = \sin(\theta) \pm \frac{\sqrt{4\sin^2(\theta) - \sin^2(2\theta)}}{2}$$

Then use trigonometric identities and simplify:

$$x = \sin(\theta) \pm \frac{\sqrt{4\sin^2(\theta) - 4\sin^2(\theta)\cos^2(\theta)}}{2}$$
$$x = \sin(\theta) \pm \sqrt{\sin^2(\theta) - \sin^2(\theta)\cos^2(\theta)}$$
$$x = \sin(\theta) \pm \sqrt{\sin^2(\theta)\left[1 - \cos^2(\theta)\right]}$$
$$x = \sin(\theta) \pm \sqrt{\sin^4(\theta)}$$
$$x = \sin(\theta) \pm \sin^2(\theta)$$

The maximum root occurs over the interval when $\sin(\theta)$ is maximized. So, using this interval, the maximum value would occur at $\frac{\pi}{2}$. Thus, the maximum possible root is $\sin\left(\frac{\pi}{2}\right) + \sin^2\left(\frac{\pi}{2}\right) = 1 + 1^2 = \mathbf{2}$.

16. Given the equation…

$$\frac{\log(0.0625)}{\log(0.\overline{1})} = \frac{\log(4^x)}{\log(9)}$$

…rewrite the equation to where you take the logarithm of the same number:

$$\frac{\log\left(\frac{1}{16}\right)}{\log\left(\frac{1}{9}\right)} = \frac{\log(4^x)}{\log(3^2)}$$
$$\frac{\log\left(\frac{1}{4}\right)^2}{\log\left(\frac{1}{3}\right)^2} = \frac{\log(4^x)}{\log(3^2)}$$
$$\frac{\log(4)^{-2}}{\log(3)^{-2}} = \frac{\log(4^x)}{\log(3^2)}$$
$$\frac{-2\log(4)}{-2\log(3)} = \frac{x\log(4)}{2\log(3)}$$
$$x = \mathbf{2}$$

17.

			1				sum=1	2^0	row 0
		1		1			sum=2	2^1	row 1
	1		2		1		sum=4	2^2	row 2
1		3		3		1	sum=8	2^3	row 3
							…	…	…
								2^{12}	row 12

Therefore, $2^{12}=(2^4)(2^4)(2^4)=(16)(16)(16)=(256)(16)=$**4096**

18. Convert 4102_5 to base-10: $2(5^0)+0(5^1)+1(5^2)+4(5^3)=527$. Then, using the stair-step division method, read the remainders downward, as shown:

$$\begin{array}{rl}
0 & \text{R1} \\
8\overline{)\,1} & \text{R0} \\
8\overline{)\,8} & \text{R1} \\
8\overline{)\,65} & \text{R7} \\
8\overline{)\,527} &
\end{array}$$

19. Each circle can intersect with each other circle a maximum of 2 times. There are $\binom{5}{2}$ ways (equal to 10) to choose a pair of circles, and each pair can intersect twice, so $(2)(10)=\mathbf{20}$ possible intersections.
20. Given $\frac{1}{\sqrt[3]{2}+\sqrt[3]{3}}$, get rid of the cubic roots by using the following method: $\frac{1}{\sqrt[3]{2}-\sqrt[3]{3}}\left(\frac{\sqrt[3]{4}+\sqrt[3]{6}+\sqrt[3]{9}}{\sqrt[3]{4}+\sqrt[3]{6}+\sqrt[3]{9}}\right)=\frac{\sqrt[3]{4}+\sqrt[3]{6}+\sqrt[3]{9}}{2-3}=\frac{\sqrt[3]{4}+\sqrt[3]{6}+\sqrt[3]{9}}{-1}$. Thus, $c=9$, and $d=-1$, so $c+d=9+-1=\mathbf{8}$.
21. Work from the inside: the complex conjugate of $3i^{14}-5i^{23}$ can be found by first simplifying to $a+bi$ form. $i^{14}=-1$, and $i^{23}=-i$, so the expression is simplified to $-3+5i$. Then, the conjugate is found by simply changing the sign in front of the bi term, so the conjugate is $-3-5i$. Then find the reciprocal by placing this over 1, and the answer is $\boldsymbol{\frac{1}{-3-5i}}$.
22. The ball will travel 6 feet plus twice (because it goes up and down) the sum of the geometric series $4+\frac{4}{3}+\frac{4}{9}+\cdots$. The sum of this series is $\frac{4}{\left(1-\frac{2}{3}\right)}=12$. So, $6+2(12)=$ $\mathbf{30}$.
23. To find intercepts, set the equation equal to zero and solve for x:

$$0=-2+\ln\left(x^2-\frac{1}{9}\right)$$

$$2=\ln\left(x^2-\frac{1}{9}\right)$$

$$e^2=x^2-\frac{1}{9}$$

$$e^2+\frac{1}{9}=x^2$$

$$\pm\sqrt{e^2+\frac{1}{9}}=x$$

Since we are looking for the positive intercept, look at the positive value: $\boldsymbol{\sqrt{e^2+\frac{1}{9}}}$.

24. Before starting the problem, put both numbers into the same base format and similar exponential format:

$$(3^4)^{z-1} = (3^4)^{x-1}$$

On the right side, we know that $4x - 1$ must be 6 in order to produce 729. So, solve for x on the right side to find what z is on the left side:

$$4x - 4 = 6$$
$$4x = 10$$
$$x = \frac{10}{4} = \mathbf{\frac{5}{2}}$$

25. Take then decimal and convert it to fractional form by performing the following:

$$\frac{p}{q} = 0.223\overline{81}$$
$$(1000)\frac{p}{q} = (0.223\overline{81})(1000)$$
$$(1000)\frac{p}{q} = 223.\overline{81} = 223 + \tfrac{81}{99} = 223 + \frac{9}{11}$$
$$(11)(1000)\frac{p}{q} = \left(223 + \frac{9}{11}\right)(11)$$
$$(11000)\frac{p}{q} = 2462$$
$$\frac{p}{q} = \frac{2462}{11000} = \frac{1231}{5500}$$

So $q - p = 5500 - 1231 = \mathbf{4269}$.

26. The circle has a radius of r units and is centered at the point (a, b). Hence, the minimum x-value is r units below the center of the circle. This point is therefore located at $(\boldsymbol{a - r}, \boldsymbol{b})$.

27. Substitute $y = 5x^2$ into the equation for the ellipse.

$$2x^2 + (5x^2)^2 = 4$$

This forms a quadratic (almost) that can be solved.

$$2x^2 + (5x^2)^2 = 4$$
$$2x^2 + (25x^4) - 4 = 0$$

Rewrite this equation into a form that is quadratic using $h = x^2$, taking into account that only the positive y-value is considered here since the parabola opens up above the x-axis:

$$25h^2 + 2h - 4 = 0$$
$$h = \frac{-2 + \sqrt{(2)^2 - (4)(25)(-4)}}{2(25)} = \frac{-2 + \sqrt{404}}{50} = \frac{-2 + 2\sqrt{101}}{50}$$
$$= 2\left(\frac{-1 + \sqrt{101}}{50}\right) = \frac{-1 + \sqrt{101}}{25}$$

Now, since h is solved for, and $h = x^2$, take the square root of h to find your answers for x:

$$h = \frac{-1+\sqrt{101}}{25}$$
$$h = x^2$$
$$x = \pm\sqrt{h} = \pm\sqrt{\frac{-1+\sqrt{101}}{25}} = \pm\frac{\mathbf{1}}{\mathbf{5}}\sqrt{\mathbf{-1+\sqrt{101}}}$$

28. First, find the number of ways to select five distinct digits:

$$\binom{10}{5} = 252$$

Then, to have five digits, you have $5! = 120$ ways to order them.
However, two orderings for each set of five distinct numbers must be thrown out since, in one way, they are strictly increasing and, in another way, they are strictly decreasing. Therefore, the total number of possible pin codes that satisfy all conditions is found by the following:

$$252(120-2) = 252(118) = \mathbf{29,736}$$

29. Solve using this template:

$$\frac{time\ together}{time\ alone_1} + \frac{time\ together}{time\ alone_2} - \frac{time\ together}{time\ alone_3} = whole\ amount - 40\%, red\ means\ drainage$$

$$\frac{t}{10} + \frac{t}{8} - \frac{t}{16} = 1 - 0.4$$
$$\frac{8t}{80} + \frac{10t}{80} - \frac{5t}{80} = 0.6$$
$$\frac{13t}{80} = \frac{3}{5}$$
$$240 = 65t$$
$$t = \frac{240}{65} = \frac{48}{13}$$
$$\frac{48}{13}\cancel{hours} * \frac{60\ minutes}{1\ \cancel{hour}} = \frac{2880}{13} = \mathbf{221\frac{7}{13}\ minutes}$$

30. First, determine what conic section the equation is:

$$y^2 + 4y - x^2 - 6x - 21 = 0$$
$$-x^2 - 6x + y^2 + 4y - 21 = 0$$
$$-(x^2 + 6x) + y^2 + 4y = 21$$
$$-(x^2 + 6x + 9) + (y^2 + 4y + 4) = 21 + 4 - 9$$
$$-(x+3)^2 + (y+2)^2 = 16$$
$$\frac{-(x+3)^2}{16} + \frac{(y+2)^2}{16} = 1$$

This is a hyperbola. So, use the formula for eccentricity of a hyperbola:

$$e = \frac{\sqrt{a^2+b^2}}{a} = \frac{\sqrt{16^2+16^2}}{16} = \frac{\sqrt{256+256}}{16} = \frac{\sqrt{512}}{16} = \frac{16\sqrt{2}}{16} = \sqrt{\mathbf{2}}.$$