For each question, "E) NOTA" indicates that none of the above answers is correct.

1. Let *A* be a 3×4 matrix, *B* be a 2×3 matrix, and *C* be a 3×2 matrix. What is the dimension of the resulting matrix product *CBA* ?

A) 3×4 B) 2×4 C) 2×3 D) 3×3 E) NOTA

2. For what value(s) of *y* is the following matrix singular? $\begin{bmatrix} y & 2 & 1 \\ 5 & 4 & 2 \\ 1 & 0 & y \end{bmatrix}$

A) $1, \frac{3}{2}$ B) $0, \frac{5}{2}$ C) $0, \frac{3}{2}$ D) $0, \frac{1}{2}$ E) NOTA

3. Given $T = \begin{bmatrix} 5 & 1 \\ 2 & 3 \end{bmatrix}$ and $Z = \begin{bmatrix} 4 & -9 \\ 6 & 7 \end{bmatrix}$, evaluate $|TZ|$.

A) 1053 B) 1120 C) 1066 D) 984 E) NOTA

4. Solve the following system of linear equations

$$\begin{aligned} 2x - 3y + z &= -2 \\ 3x - y + 2z &= -5 \\ -x + 2y - 3z &= 11 \end{aligned}$$

Express your answer as an ordered triplet (*x, y,* z).

A) (0, 0, -2) B) (-1, 0, -1) C) (0, 1, -3) D) (1, 0, -4) (E) NOTA

5. Let $A = \begin{bmatrix} 4 & -1 \\ 2 & 3 \end{bmatrix}$ and $T = \begin{bmatrix} -2 & 5 \\ 0 & 7 \end{bmatrix}$. Evaluate $(A + T)^2$.

A) $\begin{bmatrix} 2 & 44 \\ 6 & 118 \end{bmatrix}$ B) $\begin{bmatrix} 12 & 48 \\ 24 & 108 \end{bmatrix}$ C) $\begin{bmatrix} 12 & 44 \\ 24 & 118 \end{bmatrix}$ D) $\begin{bmatrix} 2 & 48 \\ 6 & 108 \end{bmatrix}$ E) NOTA

6. If *B* = $\begin{bmatrix} 2 & 1 & 0 \\ 2 & 0 & 0 \\ 2 & 0 & 1 \end{bmatrix}$, what is B^{-1}?

A) $\begin{bmatrix} 0 & \frac{1}{2} & 0 \\ -1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$ B) $\begin{bmatrix} 0 & \frac{1}{2} & 0 \\ 1 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$ C) $\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & 0 \\ 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ D) $\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & 0 \\ -1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$ E) NOTA

7. Let *C* be the adjoint of matrix *D*, where *D* is given by $\begin{bmatrix} -6 & 4 & 4 \\ 1 & -1 & -1 \\ -6 & 2 & 4 \end{bmatrix}$.
Compute the value of $((C_{31} + C_{22})C_{12})^{-C_{23}} - (C_{11}C_{32})$.

A) 3 B) 10 C) 500 D) 1000 E) NOTA

8. Given $A = \begin{bmatrix} 1 & -3 & 2 & -1 \\ -3 & 9 & -6 & 3 \\ 2 & -6 & 4 & -2 \\ -1 & 3 & -2 & 1 \end{bmatrix}$, what are the eigenvalues of *A*?

A) 0, 0, 0, 15 B) 0, 1, 1, 12 C) 0, 0, 1, 2 D) 0, 1, 2, 3 E) NOTA

9. Which of the following is NOT an eigenvector of the matrix $\begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix}$?

A) $\begin{bmatrix} 0 \\ 6 \end{bmatrix}$ B) $\begin{bmatrix} 2e \\ -e \end{bmatrix}$ C) $\begin{bmatrix} -4 \\ 2 \end{bmatrix}$ D) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ E) NOTA

10. What is the rank of $\begin{bmatrix} 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \end{bmatrix}$?

A) 5 B) 4 C) 3 D) 2 E) NOTA

11. What is the product of the eigenvalues of the matrix $\begin{bmatrix} 3 & 1 & 4 \\ 5 & 2 & 6 \\ 8 & 7 & 9 \end{bmatrix}$?

A) 0 B) 7 C) 10 D) 14 E) NOTA

12. Steven is studying the determinant of a certain 5×5 matrix *D*. He calculates $|3D| = 3402$ and $|6D| = 108864$. What is the value of $|4D|$?

A) 14336 B) 448 C) 43750 D) 4536 E) NOTA

13. Define the 2×2 matrix *A* as follows:

A_{11} = The fourth pentagonal number
A_{21} = The fifth square of a natural number
A_{12} = The sixth triangular number
A_{22} = The eight prime number

What is the value of $|A|$?

A) -297 B) -239 C) -147 D) -107 E) NOTA

14. Let $C = \begin{bmatrix} 5 & 6 & 7 \\ 8 & 9 & 10 \\ 3 & 4 & 5 \end{bmatrix}$. Chuck uses elementary row operations to obtain matrix *B*, the reduced row-echelon form of *C*. What is the value of B_{23}?

A) 2 B) 0 C) 1 D) -1 E) NOTA

15. Evaluate $\prod_{k=1}^{2} \begin{bmatrix} k+4 & 6k \\ 3k & k^3 \end{bmatrix}$

A) $\begin{bmatrix} 11 & 18 \\ 9 & 9 \end{bmatrix}$ B) $\begin{bmatrix} 66 & 108 \\ 24 & 44 \end{bmatrix}$ C) $\begin{bmatrix} 66 & 48 \\ 54 & 44 \end{bmatrix}$ D) $\begin{bmatrix} 1 & 6 \\ 3 & 7 \end{bmatrix}$ E) NOTA

16. Which of the following matrices is idempotent? A matrix is idempotent if it is equal to its own square.

A) $\begin{bmatrix} 1 & -2 & 3 \\ -4 & 5 & -6 \\ 7 & -8 & 9 \end{bmatrix}$ B) $\begin{bmatrix} 5 & -3 & 2 \\ 15 & -9 & 6 \\ 20 & -12 & 8 \end{bmatrix}$

C) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ 3 & 4 & 2 \end{bmatrix}$ D) $\begin{bmatrix} 2 & 3 & -4 \\ 0 & 1 & 0 \\ 1/2 & 3/2 & -1 \end{bmatrix}$ E) NOTA

17. Which of the following matrices is nilpotent? A matrix is nilpotent if, when raised to higher and higher powers, the results tends towards the zero matrix.

A) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ B) $\begin{bmatrix} 6 & -4 & 3 \\ 18 & -12 & 9 \\ 12 & -8 & 6 \end{bmatrix}$

C) $\begin{bmatrix} 1 & 0 & 0 \\ -2 & 3 & 0 \\ 4 & -5 & 6 \end{bmatrix}$ D) $\begin{bmatrix} 2 & 3 & -4 \\ 0 & 1 & 0 \\ 1/2 & 3/2 & -1 \end{bmatrix}$ E) NOTA

18. What is the area of a triangle whose vertices have coordinates $(5, -7)$, $(-2, 3)$, and $(4, 9)$?

A) 51 B) 102 C) 100 D) 50 E) NOTA

19. How many order pairs (x, y), where $x, y \in \mathbb{N}^+$, satisfy $\begin{vmatrix} 3x & 4y & 2 \\ 1 & 3 & 4 \\ 2 & 1 & 5 \end{vmatrix} \leq 405$?

Note: $\mathbb{N}$ denotes the set of all natural numbers.

A) 192 B) 193 C) 194 D) 195 E) NOTA

20. Let *N* be a 2×2 matrix. Given $|N| = \frac{16}{9}$, what is the value of $|(3N)^{-1}|$?

A) 16 B) $\frac{1}{16}$ C) $\frac{1}{32}$ D) $\frac{27}{16}$ E) NOTA

21. A matrix *D* is said to be a perfect square if there exists a matrix *C* such that $C^2 = D$. Which of the following matrices is a perfect square?

A) $\begin{bmatrix} 4 & 9 \\ 9 & -16 \end{bmatrix}$ B) $\begin{bmatrix} 0 & 4 \\ 4 & 4 \end{bmatrix}$ C) $\begin{bmatrix} 12 & 19 \\ 11 & 13 \end{bmatrix}$ D) $\begin{bmatrix} -6 & 16 \\ -12 & 10 \end{bmatrix}$ E) NOTA

22. Let *B* be a 2×2 matrix whose entries consist of the four natural numbers 2, 4, 6, and 8 used exactly once. If $|B| > 0$, what is the ratio of *x* to *y*, where *x* represents the smallest possible determinant of *B* and *y* represents the largest possible determinant of *B*?

A) -1 B) $\frac{2}{5}$ C) $\frac{1}{5}$ D) $\frac{1}{4}$ E) NOTA

23. When the point $(6, -4)$ is transformed according to the matrix $\begin{bmatrix} -5 & 4 \\ 3 & 2 \end{bmatrix}$, what is the resulting point?

A) $(-46, 10)$ B) $(-42, 16)$ C) $(18, 16)$ D) $(14, 10)$ E) NOTA

24. For what value of *x* are the points $(-4, 7)$, $(2, 9)$, $(12, x)$ collinear?

A) $\frac{10}{3}$ B) $\frac{25}{3}$ C) $\frac{29}{3}$ D) $\frac{37}{3}$ E) NOTA

25. Which of the following describes the matrix $\begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$?

I. symmetric
II. singular
III. square

A) I,II, and III B) II and III only C) III only D) I and III only E) NOTA

26. Using Cramer's Rule to solve a system of linear equations of three variables (*x*, *y*, *z*), Ashley determined that $y = \frac{\begin{vmatrix} -2 & 8 & -4 \\ 5 & 0 & 1 \\ 4 & 9 & -3 \end{vmatrix}}{\begin{vmatrix} -2 & 3 & -4 \\ 5 & -2 & 1 \\ 4 & 1 & -3 \end{vmatrix}}$. What is the value of $x - 3z$?

A) 1 B) 2 C) 3 D) 4 E) NOTA

27. Let *A* be a 3×3 matrix where $A_{ij} = 3i - 4j$. What is the sum of the elements in *A*?

A) -12 B) -15 C) -18 D) -21 E) NOTA

28. Consider the matrix $D = \begin{bmatrix} 2x^2 & 32x & -4x \\ x^5 & -4x & 15 \\ -7 & 6 & (x-4)^2 \end{bmatrix}$. Find all values of x such that the trace of matrix D is 7.

A) 1, 3 B) 0, 4 C) 1, -1 D) 2, 4 E) NOTA

29. Let *S* = $\begin{bmatrix} 8 & 9 & \dots & 91 \\ 16 & 18 & \dots & 182 \\ \vdots & \vdots & \ddots & \vdots \\ 448 & 504 & \dots & 5096 \end{bmatrix}$. What is the dimension of *S*?

A) 56×85 B) 55×84 C) 56×84 D) 55×85 E) NOTA

30. Using the digits 1-9 exactly once in a 3×3 matrix *A*, what is the smallest value of the determinant of *A*?

A) -404 B) -410 C) -412 D) 0 E) NOTA