Theta Perimeters, Areas, and Volumes

MA0 National Convention 2016

For each question, "(E) NOTA" indicates that none of the above answers is correct. Diagrams are not necessarily drawn to scale.

- 1. If two of the sides of an acute triangle are 14 and 48, find the greatest integral perimeter of the acute triangle.
 - (A) 111 (B) 112 (C) 123 (D) 124 (E) NOTA
- 2. Find the area enclosed by the circle inscribed in the right triangle if the circle divides the hypotenuse into segments measuring 3 m and 10 m.



6. Find the area inside a kite ABCD if $m \bigcirc ABC = 120^{\circ}, m \bigcirc DAB = 90^{\circ}, m \bigcirc BCD = 90^{\circ}, and BC = 3.$ (A) $5\sqrt{3}$ (B) $3\sqrt{3}$ (C) $\frac{9}{2}\sqrt{3}$ (D) $9\sqrt{3}$ (E) NOTA

 7. Find the area of pentagon ABCDE with vertices A(1,4),B(2,7),C(4,6),D(5,3), and E(3, -1).

 (A) 18
 (B) 24.5
 (C) 1.5
 (D) 10
 (E) NOTA

8. Find the area of DABC if the radius of the inscribed circle $\bigcirc O$ is 6 and the perimeter of DABC is 40.



9. Rectangle CNMP is inscribed in right DABC. If BC = 7, AC = 24, and AN = x, find the perimeter of rectangle CNMP in terms of x.



- 10. A rectangle has an enclosed area of 40 and a perimeter of $18\sqrt{2}$. Find the length of a diagonal of the rectangle.
 - (A) $4\sqrt{2}$ (B) $5\sqrt{2}$ (C) $\sqrt{65}$ (D) $\sqrt{82}$ (E) NOTA
- 11. Given $\bigcirc A$. If AB = 20 and BC = 20, find the length of BC.



12. Two octagons are similar. The ratio of their perimeters is 2:7. Find the ratio of their enclosed areas.

(A) 2:7 (B) 4:49 (C) 8:343 (D) 16:2401 (E) NOTA

Theta Perimeters, Areas, and VolumesMAθ National Convention 201613. Find the perimeter of a square inscribed in a circle whose radius is 3.

- (A) 18 (B) $3\sqrt{2}$ (C) $12\sqrt{2}$ (D) $12\sqrt{3}$ (E) NOTA
- 14. Two similar cones have surface areas of 32π and 50π . If the smaller cone has volume 128π , find the volume of the larger cone.
 - (A) 160π (B) 200π (C) 250π (D) $\frac{8192}{125}\rho$ (E) NOTA
- 15. The radius of $\bigcirc P$ has length 6. Find the area of the shaded sector of $\bigcirc P$.





- 16. The ratio of the circumferences of two circles is 2:5. If the area enclosed by the larger circle is 10π , find the area enclosed by the smaller circle.
 - (A) $\frac{125}{2}\rho$ (B) $\frac{8}{5}\rho$ (C) 4π (D) 25π (E) NOTA
- 17. An ice cream cone is packed solid with ice cream, and the scoop of ice cream on top of the cone is a hemisphere with diameter of 6 cm. The ice cream cone is a right cone that has a diameter of 6 cm and a height of 8 cm. Find the volume of ice cream in this ice cream cone.
 (A) 42p cm³
 (B) 60p cm³
 (C) 90p cm³
 (D) 108p cm³
 (E) NOTA
- 18. Find the volume of the solid generated by revolving the region enclosed by obtuse DABC about the altitude from A to \overrightarrow{BC} .



19. The perimeter of an isosceles right triangle is $4 + 4\sqrt{2}$. Find the length of the hypotenuse. (A) 2 (B) $2\sqrt{2}$ (C) 4 (D) $4\sqrt{2}$ (E) NOTA 20. A rectangular prism has a length of 3, a width of 4, and a height of 12. The rectangular prism is inscribed in a sphere. Find the volume of the sphere.

(A)
$$\frac{13}{2}\rho$$
 (B) $\frac{169}{3}\rho$ (C) $\frac{2197}{6}\rho$ (D) 676ρ (E) NOTA

- 21. Describe the effect on the volume of a right cylinder if the radius is divided by three and the height is doubled.
 - (A) The volume is multiplied by $\frac{1}{6}$. (B) The volume is multiplied by $\frac{2}{3}$. (C) The volume is divided by $\frac{4}{2}$. (D) The volume is divided by $\frac{9}{2}$
 - (E) NOTA
- 22. A barn with a flat roof is a rectangular prism that measures 10 yards wide, 13 yards long, and 5 yards high. Stephanie, Amy, Jeanie, and Roman are painting the barn inside and outside and on the ceiling, but not on the roof or the floor. Find the total number of square yards that they will paint.
 - (A) 490 (B) 580 (C) 720 (D) 980 (E) NOTA
- 23. A rhombus has a side of 8. The shorter diagonal of the rhombus is also 8. Find the area enclosed by the rhombus.
 - (A) $16\sqrt{3}$ (B) $32\sqrt{3}$ (C) $64\sqrt{3}$ (D) 32 (E) NOTA
- 24. Find the volume of a sphere inscribed in a cylinder with base radius of 5 and height of 10. (A) 125ρ (B) 100ρ (C) $\frac{125}{3}\rho$ (D) $\frac{500}{3}\rho$ (E) NOTA
- 25. $\bigcirc K$ and $\bigcirc C$, both with radius 12, intersect at points *H* and *W*. If *HW* = 12, find the area of the intersection of $\bigcirc K$ and $\bigcirc C$.
 - (A) $24\rho 36\sqrt{3}$ (B) $24\rho 72\sqrt{3}$ (C) $48\rho 36\sqrt{3}$ (D) $48\rho 72\sqrt{3}$ (E) NOTA
- 26. Find the volume of a pyramid whose altitude has length 4 and whose base is a rhombus with diagonals of lengths 6 and 8.
 - (A) 32 (B) 64 (C) 96 (D) 192 (E) NOTA
- 27. A cube is inscribed in a sphere. Find the ratio of the surface area of the cube to the surface area of the sphere.
 - (A) 1: ρ (B) $\sqrt{3}:\rho$ (C) 3: ρ (D) $\sqrt{6}:\rho$ (E) NOTA

28. A pie is in the shape of a right circular cylinder. The base radius of the pie is 4 in. and the height of the pie is 3 in. Hayden cuts himself a piece of pie by cutting a sector with a central angle of 40°. Find the surface area of the slice of pie.

(A)
$$\left(24 + \frac{44}{9}\rho\right) in^{2}$$
 (B) $\left(12 + 8\rho\right) in^{2}$ (C) $\left(12 + \frac{28}{9}\rho\right) in^{2}$
(D) $\left(24 + \frac{56}{9}\rho\right) in^{2}$ (E) NOTA

- 29. Bradley wants to set a new record by eating the world's largest chocolate bar. The chocolate bar is in the shape of a right square pyramid with a base edge that measures 8 feet. Bradley begins eating from the top, finishing a portion of the chocolate bar. The remaining chocolate is a frustum of the original pyramid. The height of the frustum is 12 feet and the upper base edge measures 6 inches. If the lower base has remained the same and no chocolate was removed between the upper and lower bases of the frustum, how many more cubic feet of chocolate will Bradley need to eat?
 - (A) 273 ft³ (B) 592 ft³ (C) $\frac{2045}{7}$ ft³ (D) $\frac{2045}{7}$ ft³ (E) NOTA
- 30. The base of a right prism is a regular octagon. The area enclosed by the octagon is 26. The altitude of the prism has length 15. Find the volume of the prism.

(A) 130	(B) 260	(C) 390	(D) 520	(E) NOTA