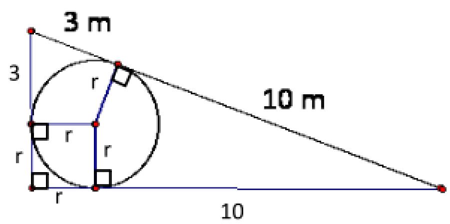
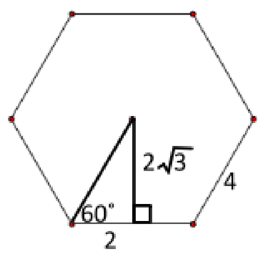
1. A The sum of two sides of a triangle must be greater than the 3rd side. 14 + 48 > *x*, *x* < 62. In an acute triangle the square of the longest side must be less than the sum of the squares of the other two sides. $x^2 < 14^2 + 48^2$, *x* < 50. The largest integer less than 50 is 49. The perimeter of the triangle is 111.

2. D



Tangents that go to the same circle from the same exterior point are congruent.
Tangents are perpendicular to radii at the points of tangency.
Pythagorean Theorem:
$(3 + r)^2 + (r + 10)^2 = 13^2$
$r^2 + 13r - 30 = 0$
r = 2 (r = –15 is extraneous)
$A = \pi r^2 = 4\pi$ m^2

3. D



$A = \frac{a \cdot P}{2}$

$A = \frac{2\sqrt{3} \cdot 24}{2}$

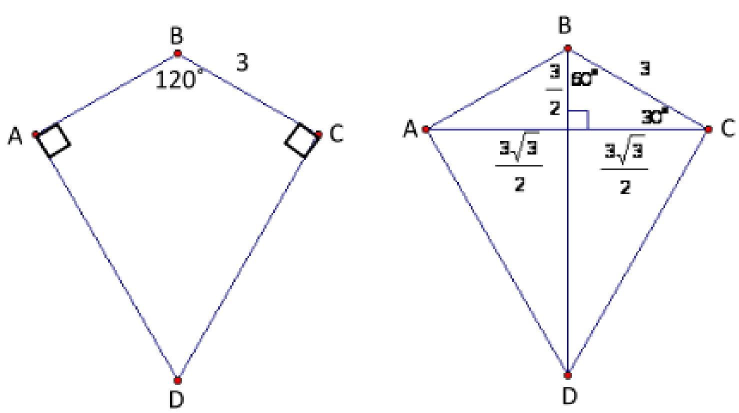
$A = 24\sqrt{3}$

4. B When base angles of a triangle are congruent, the sides opposite the angles are also congruent.

$4x^2 + 1 = 6x + 1$. $x = \frac{3}{2}$ (*x* = 0 is extraneous) . Substitute $\frac{3}{2}$ in for x to find that the perimeter is 111.

5. A $AE^2 + EB^2 + DE^2 + EC^2 = \text{diameter}^2$ $\text{diameter} = 5\sqrt{10}$ $C = pd$ $C = 5p\sqrt{10}$

6. D



BD = 6, AC = $3\sqrt{3}$

Area = $\frac{d_1 \cdot d_2}{2}$

Area = $9\sqrt{3}$

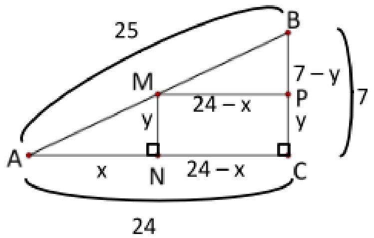
7. A

$$\frac{\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \\ x_5 & y_5 \\ x_1 & y_1 \end{vmatrix}}{2} = \frac{\left| x_1y_2 + x_2y_3 + x_3y_4 + x_4y_5 + x_5y_1 - x_1y_5 - x_5y_4 - x_4y_3 - x_3y_2 - x_2y_1 \right|}{2}$$

Area = 18

8. D $\text{Area} = \text{radius} \times \text{semiperimeter} \quad A = 6 \times 20 \quad A = 120$

9. C



ΔAMN ~ ABC by Angle-Angle Similarity Postulate.

$\frac{x}{24} = \frac{y}{7}$

$y = \frac{7}{24}x$

$2\left(\frac{7}{24}x\right) + 2(24 - x)$

$48 - \frac{17}{12}x$

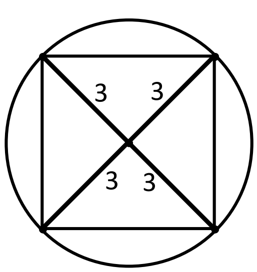
10. D $\text{Area} = \ell \cdot w \quad \text{Perimeter} = 2\ell + 2w \quad \begin{matrix} \ell \cdot w = 40 \\ 2\ell + 2w = 18\sqrt{2} \end{matrix}$ Solve the system of equations. The sides of the rectangle are $4\sqrt{2}$ and $5\sqrt{2}$. The length of the rectangle's diagonal can be found using the Pythagorean Theorem. $(4\sqrt{2})^2 + (5\sqrt{2})^2 = \text{diagonal}^2$. The diagonal measures $\sqrt{82}$.

11. C $AC = 12$ since all radii in the same circle are congruent. Thus, ΔABC is equilateral and $m\angle A = 60°$. $\text{Length of an arc} = \frac{\text{measure of the arc} \times 2\pi r}{360°}$.

Length of $\overset{\frown}{BC} = \frac{60° \times 2\pi 12}{360°} = \frac{20}{3}\pi$.

12. B If the ratio of the perimeters is 2:7, then the scale factor is also 2:7. If the scale factor is 2:7, then the ratio of the areas is $2^2 : 7^2$, or 4:49

13. C



The diagonals of a square are congruent, they bisect each other, and they are perpendicular to each other. The side of the square measures $3\sqrt{2}$. Thus the perimeter of the square is $12\sqrt{2}$.

14. C $\dfrac{\text{SA smaller cone}}{\text{SA larger cone}} = \dfrac{32\pi}{50\pi} = \dfrac{16}{25}$ If the ratio of the surface areas is $\dfrac{16}{25}$, then the scale factor is $\dfrac{\sqrt{16}}{\sqrt{25}} = \dfrac{4}{5}$. If the scale factor is $\dfrac{4}{5}$, then the ratio of the volumes is $\dfrac{4^3}{5^3} = \dfrac{64}{125}$ $\dfrac{64}{125} = \dfrac{128p}{V_L}$

$V_L = 250p$

15. A If the minor arc measures 42˚, then the major arc measures 318˚.

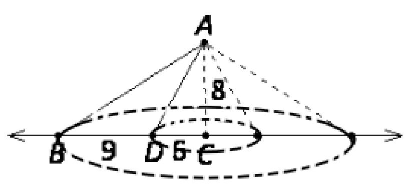
Area of the sector = $\dfrac{\text{measure of the arc} \cdot \pi r^2}{360°} = \dfrac{318° \cdot \pi \cdot 6^2}{360°} = \dfrac{159}{5}\pi$

16. B If the ratio of the circumferences of two circles is $\dfrac{2}{5}$, then the ratio of the areas of the circles is

$\dfrac{2^2}{5^2} = \dfrac{4}{25}$. $\dfrac{\text{area of the smaller circle}}{\text{area of the larger circle}} = \dfrac{4}{25} = \dfrac{A_S}{10p}$ $A_S = \dfrac{8}{5}p$

17. A Volume of the ice cream = $V_{\text{Hemisphere}} + V_{\text{Cone}} = \dfrac{2\pi r^3}{3} + \dfrac{\pi r^2 h}{3} = \dfrac{2\pi 3^3}{3} + \dfrac{\pi 3^2 \cdot 8}{3} = 42\pi$

18. B



Volume = $V_{\text{larger cone}} - V_{\text{smaller cone}}$

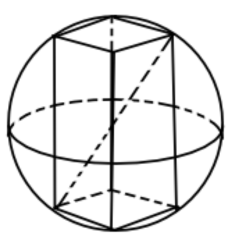
Volume = $\dfrac{p r_L^2 h_L}{3} - \dfrac{p r_S^2 h_S}{3}$

Volume = $\dfrac{p 15^2 \times 8}{3} - \dfrac{p 6^2 \times 8}{3}$

Volume = $504p$

19. C $x + x + x\sqrt{2} = 4 + 4\sqrt{2}$ $x = 2\sqrt{2}$ The hypotenuse measures 4.

20. C

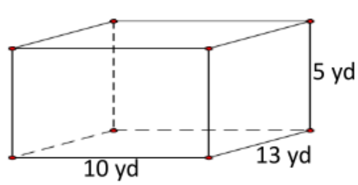


The space diagonal of the rectangular prism is the diameter of the sphere. diameter = $\sqrt{\ell^2 + w^2 + h^2} = \sqrt{3^2 + 4^2 + 12^2} = 13$

Volume of the sphere = $\dfrac{4\pi r^3}{3} = \dfrac{4\pi\left(\dfrac{13}{2}\right)^3}{3} = \dfrac{2197}{6}\pi$

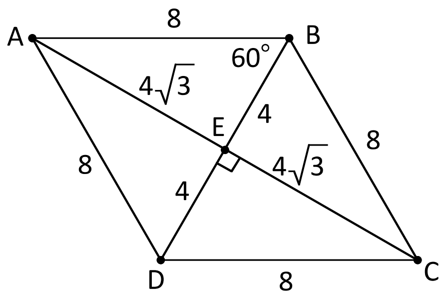
21. D Volume of a right cylinder = $\pi r^2 h$ $\pi(3)^2 \times 2$ multiply by $\dfrac{2}{9}$ or divide by $\dfrac{9}{2}$

22. E



They will paint the inside and outside of each wall (lateral face of the prism): $4 \times 10 \times 5 + 4 \times 5 \times 13$. They will paint the ceiling (base of the prism): 10×13. The total number of square yards that they will paint is 590 yd^2.

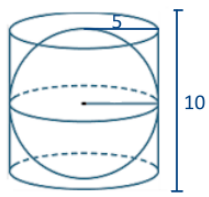
23. B



Since $\overline{AB}$, $\overline{AD}$, and $\overline{BD}$ all measure 8, ΔABD is an equilateral triangle and $m\angle ABD = 60^\circ$. The diagonals of a rhombus are perpendicular to each other and they bisect each other.

Area of a rhombus = $\frac{d_1 \cdot d_2}{2} = \frac{8 \cdot 8\sqrt{3}}{2} = 32\sqrt{3}$

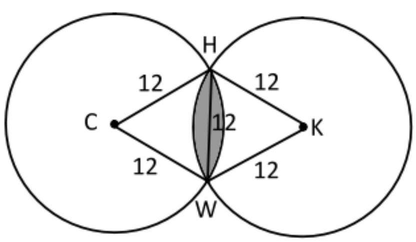
24. D



The height of the cylinder is equivalent to the diameter of the sphere. Since the height is 10, the radius of the sphere is 5.

Volume of a sphere = $\frac{4\pi r^3}{3} = \frac{4\pi 5^3}{3} = \frac{500}{3}\pi$

25. D



Find the area of the segment of $\odot C$ bounded by the chord $\overline{HW}$ and $\overset{\frown}{HW}$.

$$\text{Area of the segment of } \odot C = \frac{\text{measure of the arc} \times \pi r^2}{360^\circ} - \frac{r^2\sqrt{3}}{4}$$

$$\text{Area of the segment of } \odot C = \frac{60^\circ \pi \times 12^2}{360^\circ} - \frac{12^2\sqrt{3}}{4}$$

$$\text{Area of the segment of } \odot C = 24\pi - 36\sqrt{3}$$

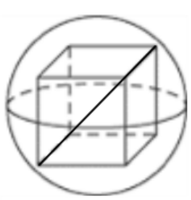
$$\text{Area of the segment of } \odot C = \text{Area of the segment of } \odot K$$

$$\text{Area of the shaded region} = 48\pi - 72\sqrt{3}$$

26. A Volume Pyramid = $\frac{Bh}{3}$ The base is a rhombus with diagonals measuring 6 and 8.

$B = \frac{d_1 \times d_2}{2} = \frac{6 \times 8}{2} = 24$ $V = \frac{24 \times 4}{3} = 32$

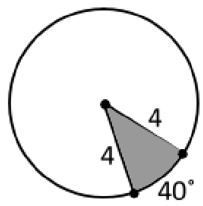
27. E



Since the cube is inscribed in the sphere, the space diagonal of the cube is the diameter of the sphere. Space diagonal = $s\sqrt{3}$, where s represents the edges of the cube. The radius of the sphere, r, is half the length of the space diagonal. $r = \frac{s\sqrt{3}}{2}$

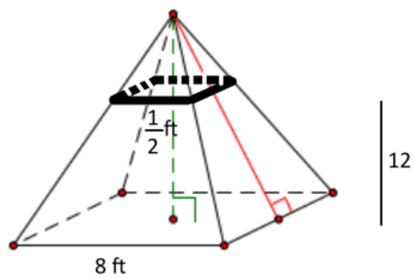
$$\frac{\text{surface area of the cube}}{\text{surface area of the sphere}} = \frac{6s^2}{4\pi r^2} = \frac{6s^2}{4\pi\left(\frac{s\sqrt{3}}{2}\right)^2} = \frac{2}{\pi}$$

28. D



The piece of cake is a sector of the cylinder. Two of the lateral faces are rectangles: $2\times 4\times 3 = 24$, the curved lateral surface: $\frac{40°\cdot 2\pi 4\cdot 3}{360°} = \frac{24}{9}\pi$, the bases are sectors of the circle: $\frac{40°\pi 4^2}{360°} = \frac{32}{9}\pi$. The surface area of the slice of cake is $\left(24 + \frac{56}{9}\pi\right)$ in^2.

29. A



$$\text{Volume of a frustum} = \frac{1}{3}h\left(B_1 + B_2 + \sqrt{B_1 B_2}\right)$$

B_1 = area of larger base of the frustum

B_2 = area of smaller base of the frustum

h = height of the frustum

$$\text{Volume of frustum} = \frac{1}{3}12\left(8^2 + \left(\frac{1}{2}\right)^2 + \sqrt{8^2\left(\frac{1}{2}\right)^2}\right)$$

$$\text{Volume of frustum} = 4\left(64 + \frac{1}{4} + 4\right)$$

$$\text{Volume of frustum} = 273$$

30. C Volume of octagonal prism = area of the base × height of the prism = $26\times 15 = 390$.