"For all questions, answer choice "E. NOTA" means none of the above answers is correct."

1. The sum of the integers 1 through n can be modeled by a quadratic polynomial. What is the product of the non-zero coefficients of that polynomial?

A. 0 B. ½ C. 1 D. 2 E. NOTA

Answer B. The formula is $\frac{n(n+1)}{2}$ which has two coefficients of ½, so the product is ¼.

2. What is the sum of the x-intercepts of $g(t) = e^{2t} - e^{t+ln9} + 14$

A. ln2 B. $\ln(\frac{2}{7})$ C. ln7 D. ln14 E. NOTA

Answer D. The substitution $x = e^t$ yields a factorable quadratic in x for which the roots are 2 and 7. So the x-intercept occur at t = ln2 and t = ln7, the sum of which is D.

3. What is the value of $\frac{1}{2+\frac{1}{2+\frac{1}{2+\cdots}}}$?

A. $-1-\sqrt{2}$ B. $\frac{1}{3}$ C. $\sqrt{2}$ D. $-1+\sqrt{2}$ E. NOTA

Answer D. Assigning x to the value of the expression and substituting for the second term in the first denominator (which is an equivalent expression) yields the quadratic $x^2 + 2x - 1 = 0$ which has both A and D as solutions, but A is extraneous since it is negative.

4. If $f(x) = ax^2 + bx + c$ and the points (0, 8) and (1, 9) are on $f(x)$, what is the value of $a + b + c$?

A. 1 B. 8 C. 9 D. 10 E. NOTA

Answer C. $f(1) = a + b + c = 9$

5. What is the equation of the tangent line to the function $f(x) = x^2$ at (2,4)?

A. $y = \frac{5}{2}x - 1$ B. $y = 3x - 2$ C. $y = \frac{7}{2}x - 3$ D. $y = 4x - 4$

E. NOTA

Answer D. The focus of a parabola forms an isosceles triangle with the point of tangency and the intersection point of the tangent line and the axis of the parabola (with the focus being the vertex and the tangent line forming the base). Using this we can calculate the

distance from the focus to (2,4) and that would be the same as the distance from the focus to the third point of the triangle on the axis. Using the formula $x^2 = 4py$ we can determine that the focus is $(0, \frac{1}{4})$, making that distance $\frac{17}{4}$. This makes the third point $(0, -4)$. The equation of the tangent line is then the equation of the line passing between that point and (2,4).

6. If $f(x) = ax^2 + bx + c$ and it's vertex is $(5, 6)$, what is the value of $\frac{a}{b}$?

A. -10 B. $-\frac{2}{5}$ C. $-\frac{1}{12}$ D. $\frac{1}{10}$ E. NOTA

Answer E. The x-coordinate of the vertex is $-\frac{b}{2a}$. Therefore multiplying by -2 and reciprocating will yield $-\frac{1}{10}$.

7. If $121_b = 256$, what is the value of b?

A. 12 B. 13 C. 14 D. 15 E. NOTA

Answer D. 121_b is equivalent to $(b+1)^2$ therefore b=15.

8. If a ball is thrown straight up with an initial velocity of 4 ft./sec. from a height of 16 ft. above the ground, how long will it take to hit the ground (approximate the acceleration due to gravity on Earth as $-32\ ft./\text{sec.}^2$)?

A. 1.25 B. $\frac{3+\sqrt{41}}{8}$ C. $\frac{3+2\sqrt{3}}{4}$ D. 2 E. NOTA

Answer D. The formula for projectile motion is the quadratic $y(t) = -\frac{g}{2}t^2 + v_0 t + h_0$. Substituting the appropriate values from the information given yields the equation: $y(t) = -16t^2 + 24t + 16$. Then we simply need to set this equal to zero, giving us $-\frac{1}{2}$ and 2 as solutions. Since the ball cannot hit the ground before it is thrown, the former is extraneous.

9. $f(x) = ax^2 + 2x + 1$ and $g(x) = x^2 + 6x + 2$. If $a < 0$, then for what value of a is there only one intersection point between f(x) and g(x)?

A. -4 B. -3 C. -2 D. -1 E. NOTA

Answer B. The intersection point(s) of two functions can be found by setting them equal to each other. The resulting quadratic (after combining like terms) is $(1-a)x^2 + 4x + 1 = 0$. If this is to have only one solution the discriminant, $16 - 4(1-a)$, must be equal to zero, therefore $a = -3$.

10. What are the coordinates of the focus of $f(x) = 4x^2 + 16x + 17$?

Answer B. After rewriting in vertex form it is obvious the vertex is at $(-2, 1)$. Given that the a value is 4, the p value is $\frac{1}{16}$, and since the parabola is facing upward the focus is one sixteenth of a unit above the vertex.

11. What is the sum of the zeroes of $f(x) = \left(1 + \sqrt{3}\right)x^2 + 528\pi x + e^3$

A. $\pi(264 - 264\sqrt{3})$ B. $\pi(132 - 132\sqrt{3})$ C. $\pi(132\sqrt{3} + 132)$
D. $\pi(264\sqrt{3} + 264)$ E. NOTA

Answer A. The sum of the zeroes of any quadratic is $-\frac{b}{a}$. After rationalizing, you get A.

12. If a right triangle has an area of 17 one leg that is twice the length of the other, what is the length of the shorter leg?

A. $\frac{\sqrt{34}}{2}$ B. $\sqrt{17}$ C. $\sqrt{34}$ D. $2\sqrt{17}$ E. NOTA

Answer D. If x is the short leg then we get the equation $17 = \frac{1}{2}(x)(2x)$, the solution should be obvious from here.

13. Find the product of all solutions of $\sqrt{x^2 + 3x - 40} + \sqrt{2x^2 + 7x - 52} = 0$

A. -92 B. -24 C. 24 D. There are no solutions E. NOTA

Answer D. Moving the second term to the other side gives an equation between a positive and negative radical. These can only be equal if they share zeroes, and these do not. Both solutions generated from squaring both sides are extraneous.

14. A farmer wishes to construct a fenced area alongside a river of width w and length l. If he needs no fence on the side with the river (i.e. only needs fence along three sides) and has 100 ft. of fence to work with and wants to enclose the largest area he can, what is the value of $w + l$?

A. 50 B. $\frac{200}{3}$ C. 70 D. 75 E. NOTA

Answer D. Based on the above, $l = 100 - 2w$ which means $A = -2w^2 + 100w$, the maximum of which occurs at $w = 25$, making $l = 50$.

15. If the line y = 2 intersects a parabola at two points a distance of 2 apart and one of those points is (2, 2), what is the product of the two possible x-coordinates for the vertex of the parabola?

A. -2 B. 0 C. 2 D. 4 E. NOTA

Answer E. Since it is a horizontal line, the axis of the parabola acts as a perpendicular bisector to it, therefore the axis (and consequently the x-coordinate of the vertex) should be 1 unit away from (2, 2) either at $x = 1$ or $x = 3$, making the product 3.

16. If $0 \leq t \leq 2\pi$ what is the sum of all possible values of t (in radians) that satisfy the following equation: $4sin^2t - 3 = 0$

A. 0 B. π C. 2π D. 4π E. NOTA

Answer D. Solving the quadratic for $sin(t)$ yields that it can equal $\pm\frac{\sqrt{3}}{2}$, therefore t can be any of the four $\frac{\pi}{3}$ multiples in the interval, the sum of which is 4π.

17. $f(x) = ax^2 + bx + 3$ and has exactly one zero at $x = \frac{1}{2}$. If $a \neq 0$, find ab.

A. -144 B. $-\frac{9}{4}$ C. $\frac{9}{4}$ D. 144 E. NOTA

Answer A. If a parabola has only one zero it must occur at the vertex and the discriminant of the parabola must be zero. This produces the following system: $-\frac{b}{2a} = \frac{1}{2}$ and $b^2 - 12a = 0$. Solving the first equation gives us $b = -a$ and substituting into the second we get $a^2 - 12a = 0$, the solutions for which are 12 and 0, but the latter is excluded in the conditions of the problem. This means $b = -12$ and hence the answer A.

18. Which of the following best describes the conditions under which the statement $(a + b)^2 = a^2 + b^2$ must be true?

A. $a = 0$ B. $b = 0$ C. It is always true D. Either A or B E. NOTA

Answer D. The difference is the middle term $2ab$ which is equal to zero when either a or b is zero, hence answer D.

19. $f(x) = a(x - h)^2 + k$ contains the points $(1, 0), (2, 2), (3, 8), (4, 18), and\ (5, 32)$. Find the value of $(a - h)k$.

A. -1 B. 0 C. 1 D. 3 E. NOTA

Answer B. The pattern in the y-coordinates of the points given is that of double each perfect square. This makes it fairly obvious that $a = 2$. From there deducing that the first set of coordinates represents the vertex is not difficult giving us $k = 0$. One could also plug in three points and solve the resulting system, but that is a bit more tedious.

20. Which of the following is NOT a Pythagorean triple?

A. 13-84-85 B. 23-264-265 C. 31-480-481 D. 41-840-841 E. NOTA

Answer E. Pythagorean triples can easily be generated by noting that if the smallest side is odd and the other two sides are one apart, the square of the smallest side is equal to the sum of the other two sides. This is the case in all four answers.

21. Which of the following is the correct geometric definition of a parabola?

A. The set of all points equidistant from a given point
B. The set of all points equidistant from two given points
C. The set of all points equidistant from a given point and a line
D. The set of all points the sum of whose distances from two given points is constant
E. NOTA

Answer C. The first is a circle, the second is a line, and the fourth is an ellipse.

22. Let $f(x) = ax^2$ and $f^n(x)$ denote n compositions of f with itself, such that $f^2(x) = f\big(f(x)\big),\ f^3(x) = f\big(f(f(x))\big)$, and so on. Find $f^9(x)$

A. a^9x^{18} B. a^9x^{512} C. $a^{511}x^{18}$ D. $a^{511}x^{512}$ E. NOTA

Answer D. The power of x is doubled each time so 2^9 will be the power of x (which is 512). The power of a is also doubled each time, but then has one more added to it (from the multiplication) the resulting pattern is one below successive powers of 2, making the power of a $2^9 - 1$ which is 511.

23. If a parabola has a vertex at $(3, 5)$ and a focus at $(3, 3)$ what is the equation of the parabola?

A. $y = -8(x-3)^2 + 5$ B. $y = -2(x-3)^2 + 5$ C. $y = -\frac{1}{2}(x-3)^2 + 5$
D. $y = -\frac{1}{8}(x-3)^2 + 5$ E. NOTA

Answer D. The equation is $4p(y-k) = (x-h)^2$ where (h, k) is the vertex and p is the distance from the vertex to the focus (and negative if the focus is below the vertex).

24. If sphere A has 64 times the surface of sphere B, how many times larger is the radius of sphere A than that of sphere B?

A. 4 B. 8 C. 64 D. 4096 E. NOTA

Answer B. The surface area of a sphere is $4\pi r^2$ so if a radius is 8 times larger than another the corresponding surface area will be 64 times larger.

25. What is the sum of the y-coordinates of the intersection points of $f(x) = 2x^2 - 5x + 8$ and $(x) = 3x^2 - 7x - 72$?

A. -2 B. 96 C. 158 D. 254 E. NOTA

Answer D. Set the equations equal to each other to find 10 and -8 as the solutions for x. Plugging these in yields 158 and 176 as the corresponding y-coordinates respectively.

26. $(a - 2b + 3c - 4d)^2 - (4d - 3c + 2b - a)^2 = ?$

A. 0 B. $2(a - 2b + 3c - 4d)$ C. $2(a + 2b + 3c + 4d)$ D. $2(a - 2b + 3c - 4d)^2$ E. NOTA

Answer A. The two expressions in parentheses are additive inverses. Squaring them yields equivalent expressions which are being subtracted from one another.

27. Which of the following can be factored as a "difference of two square numbers"?

A. $2016g + 25g^2 + 1$
B. $2015^2 - g^2$
C. $201{,}600^2 + 100g$
D. $201{,}600^2 + 20{,}160g$
E. NOTA

Answer B. This answer is a difference of squares which yields the expression asked for. Each other answer is missing something or is slightly off.

28. If $f(x) = (x + 1)^2$ and $f(a + b) = 2016$ and $f(a - b) = 1620$ what is $ab + b$?

A. 99 B. 198 C. 396 D. 909 E. NOTA

Answer A. Treating each trinomial square as a binomial square with $(a + 1)$ as one of the terms yields the following two equations: $(a + 1)^2 + 2(a + 1)b + b^2 = 2016$ and $(a + 1)^2 - 2(a + 1)b + b^2 = 1620$. Subtracting the second from the first leaves us with $4(a + 1)b = 396$. From here simply divide by 4 on both sides.

29. Give the equation of the line that intersects the following parabola at its vertex and y-intercept: $f(x) = 3x^2 - 12x + 8$

A. $y = -13x + 8$ B. $y = -12x + 8$ C. $y = -6x + 8$ D. $y = -2x + 8$
E. NOTA

Answer C. The y-intercept is $(0, 8)$ and the vertex is $(2, -4)$. This gives us a slope of -6 and the same y-intercept.

30. In terms of p, the distance from the focus to the vertex, what is the area of the isosceles triangle which has the *latus rectum* as the base and the vertex of the parabola as the opposite vertex of the triangle?

A. $\frac{1}{2}p$ B. $\frac{1}{2}p^2$ C. p^2 D. $2p^2$ E. NOTA

Answer D. The length of the *latus rectum* in terms of p is 4p (this can be proved by plugging in p for y in abstract and solving for x). Therefore 4p is the base and p is the height.